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On the Optimization of Decimation Filter in the Short Time DFT Based on Remez Algorithm

Remezアルゴリズムに基づくShort Time DFTのデシメーションフィルタの最適化

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ABSTRACT *Decimation filters of the ST gDFT Hilbert transformer are examined through frequency response with employing Remez algorithm. Optimization is performed alternately on the frequency and time domain with restricting filter coefficients. This optimized response of the Hilbert transformer also defines the optimized weighting function for the decimation filters. In other words, the optimized Hilbert transformer response based on the Remez algorithm deduces the optimizing weighting function of the significant decimation filter in the ST DFT.*

1. INTRODUCTION

The instantaneous spectrum in so important concept that the previously reported short time DFT(ST DFT) Hilbert transformer is realized to be almost free from any error both in phase shifting and amplitude owing to employing shifting the phase on the phase plane [1]. This concept is provided with ST DFT to make many applications being feasible in such radio communication systems as high speed MODEM, highly efficient CODEC, and distortion free filters in addition to the Hilbert transformer. The significant functions in the ST DFT are mainly characterized with the decimation filters which a playa important role during analyzing input signals [2].

In this paper, we discuss about optimization of the decimation filters used in the generalized short time DFT(ST gDFT) Hilbert transformers with employing Remez Algorithm [3]. Optimization is performed alternatively on the frequency and time domains with adjusting time domain response, i.e. unit sample response. The unit sample response optimized in frequency domain is transformed via inverse ST gDFT (ST gIFT) to define the coefficients of the decimation filters [4].

So long as this optimized Hilbert transformer is realized ideally, the optimized unit sample response of the ST gDFT Hilbert transformer also suggests the optimum weighting function for the decimation filter in a ST DFT, which is also universally adopted to filter bank systems.

As discussed details in following session, the ideal Hilbert transformer is realized with employing the relation $\widehat{G}(\omega) = -j\,sign\,\omega\,G(\omega)$, here $\widehat{G}(\omega)$ is Hilbert transform of the Fourier transform of the arbitrary function $g(t)$. The ST gDFT guarantees the solver for avoiding singularity around zero frequency.

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Therefore, the optimized ST gDFT Hilbert transformer consequently gives the newly proposing optimization algorithm for the significant decimation filters in the analysis of the instantaneous spectrum.

2. PRINCIPLE OF THE ST gDFT

The previously reported ST DFT Hilbert transformer is provided with causality based on restricting the 0_{th} and $\frac{N}{2}th$ sub-channels being null [1]. Nullification on these sub-channel reduces the bandwidth over subjective domain as shown in fig.1(b).

Fortunately, the ST gDFT is able to release

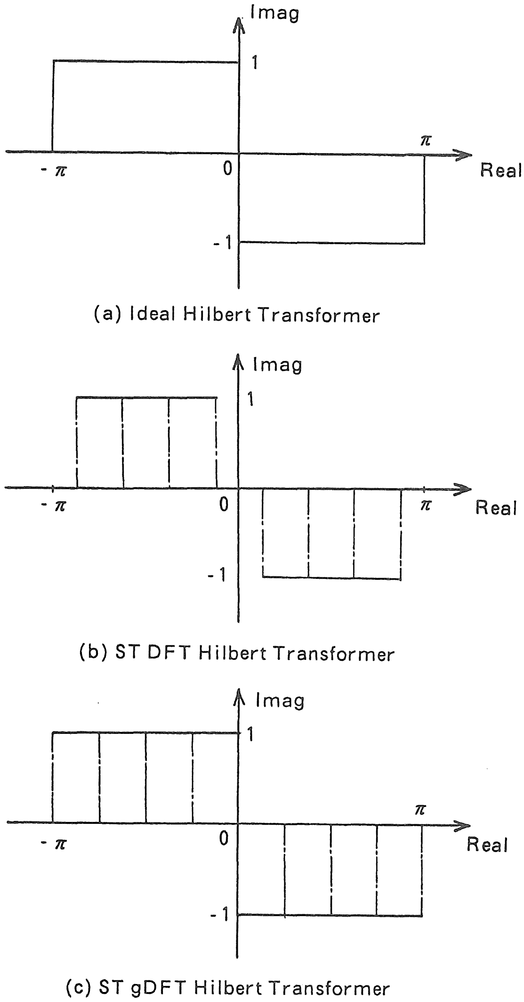

Fig.1 Comparison of the passband among ideal(a), ST DFT(b) and ST gDFT Hilbert transformers(c).

this restriction from causality. As shown in fig.1(c), the ST gDFT is able to adjust its channel allocation to avoid zero cross in the 0_{th} sub-channel in order to coincide with that of ideal Hilbert transformer shown in fig.1(a). That is, the instantaneous spectrum $\phi_k(n)$ is given by ST gDFT as follows.

$$\phi_k(n) = \sum_{r=-\infty}^{\infty} h(n-r)\,x(r)W_N^{-(k+\xi)r}, \quad 0 \le k < N \qquad (1)$$

where $h(*)$ is an apriori decimation filter ;$x(r)$ is a sampled data at time r; $W_N^{-(k+\xi)r}$ are ST gDFT operators,

$$W_N^{-(k+\xi)r} = e^{-j\frac{2\pi}{N}(k+\xi)r}, \quad 0 \le \xi < 1 \qquad (2)$$

here, ξ is newly introduced parameter to adjust channel allocation of the existing ST DFT.

Where the parameter ξ is set to be $\frac{1}{2}$, the channel allocation of the ST gDFT is moved up by half of sub-channel width to coincide 0_{th} lower fringe with zero frequency. Attentions must be paid on this channel allocation to get ideal Hilbert transformer based on the phase place relation $\widehat{G}(\omega) = -j\,sign\,\omega\,G(\omega)$, and on that bandwidth of realized Hilbert transformer will become to equal to that of ideal one as shown in fig.2 because there exist no sub-channel eliminations in the ST gDFT Hilbert transformer.

In fig.2, solid curve shows amplitude frequency response of the ST gDFT Hilbert trans-

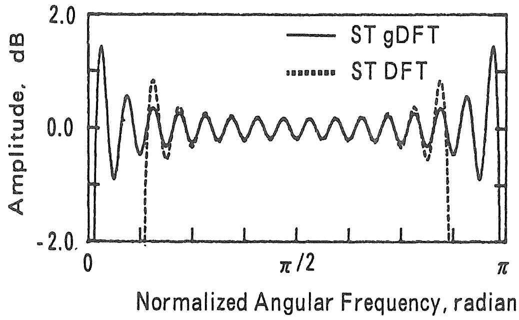

Fig.2 Comparison of amplitude frequency response between ST gDFT and ST DFT Hilbert transformers(2m=8, N=8).

former and dotted curve shows that of the ST DFT one. Wideness of the ST gDFT Hilbert transformer is shown clearly in the figure if both window length $2m = 8$ and frame length $N = 8$.

3. DECIMATION FILTERS IN THE ST DFT AND ST gDFT

3.1. Definition of the Decimation Filters

Necessary condition for the decimation filter defined by $h(*)$ in eq.1 is deduced from specification as an ideal low-pass filter,

$$N(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{N} \le \omega \le \frac{\pi}{N} \\ 0, & else. \end{cases} \quad (3)$$

Inverse Fourier transform for $N(e^{j\omega})$ gives impulse response of the decimation filter $n(n)$ so called Nyquist,

$$n(n) = \frac{1}{T}\int_{-\frac{\pi}{N}}^{\frac{\pi}{N}} H(e^{j\omega})\, e^{j\omega n} d\omega = \frac{\sin(n\pi/N)}{n\pi/N} \quad (4)$$

As shown clearly in eq.4, infinite frame number Nyquist behaves as an ideal decimation filter with fatal victim of paying infinite processing amount during convolution. When the Nyquist function is truncated by finite length, amplitude peak values both of main and side lobes becomes to be greater, and the sharpness of cut-off becomes to be vague to show Gibbs's phenomenon on the frequency domain.

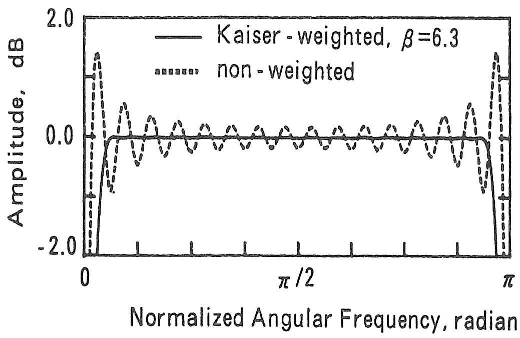

Fig.3 Comparison of amplitude frequency response betweeen Kaiser-weighted and non-weighted ST gDFT Hilbert transformers (2m=8, N=8).

Let's consider what effect will be introduced by weighting the truncated Nyquist by Kaiser function as follows.

$$h(n) = n(n)k(n), \quad (5)$$

where,

$$k(n) = \frac{I_0\left(\beta\sqrt{1-\left(\frac{n}{mN}\right)^2}\right)}{I_0(\beta)}, \quad (6)$$

$$-mN \le n \le mN$$

here $I_0(*)$ is the modified 0_{th} order Bessel of the first kind, β is an arbitrary positive real number to adjust the width and energy of 0_{th} the mainlobe.

Solid curve in fig.3 shows the amplitude frequency response of the ST gDFT Hilbert transformer adopted with Kaiser weighted Nyquist of eq.5, here frame length $N = 8$, and frame number $2m = 8$, and $\beta = 6.3$. Dotted curve in fig.3 simultaneously shows the amplitude frequency response of the ST gDFT Hilbert transformer in which the decimation filter $h(*)$ is adopted with Nyquist merely truncated by $2m$ frame number. It is clearly shown that the amplitude error over subjective domain $(0, \pi)$ is remarkably improved by employing Kaiser weighting function given by eq.6.

Figure 4 shows that the maximum amplitude error on the subjective domain of the ST gDFT Hilbert transformer adopted Kaiser weighted

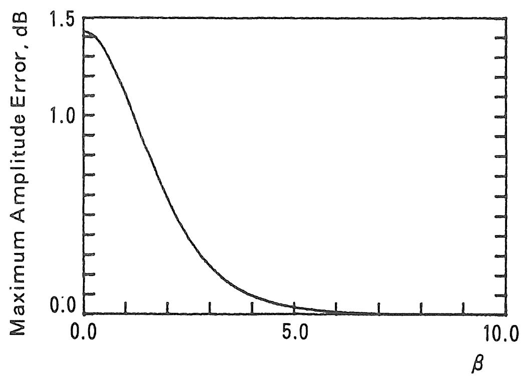

Fig.4 Maximum amplitude error of the ST gDFT Hilbert transformer with Kaiser-weighted Nyquist vs. β (2m=8, N=8).

Nyquist is monotonously improved as the parameter β goes large, and is improved to be less than 0.01dB if β is greater than 6.3.

Contradictorily, the bandwidth of the transformer is slightly shrinked from increasing the value of β as shown in fig.5. If β is set to be 10, the bandwidth is shrinked by 4.5 point by percent. Kaiser weighted Nyquist is consequently recognized as a suitable decimation filter in the meanings both of minimum amplitude error and maximum bandwidth under restriction condition of β =6.3 as indicated in figs.4 and 5.

4. OPTIMIZATION OF THE DECIMATION FILTERS

4.1 Optimization of the ST gDFT Hilbert Transformer

The frequency response of the ST gDFT Hilbert transformer $H(e^{j\omega})$ of length $2M+1$ is given as

$$H(e^{j\omega}) = \sum_{n=-M}^{M} h(n)\, e^{-j\omega n}, \quad M = mN. \tag{7}$$

The causality is well known given by merely adding delay by Msamples. $H(e^{j\omega})$ is also modified from the symmetry of $h(n)$ as follows.

$$H(e^{j\omega}) = h(0) + \sum_{n=1}^{M} 2h(n)\cos(\omega n) \tag{8}$$

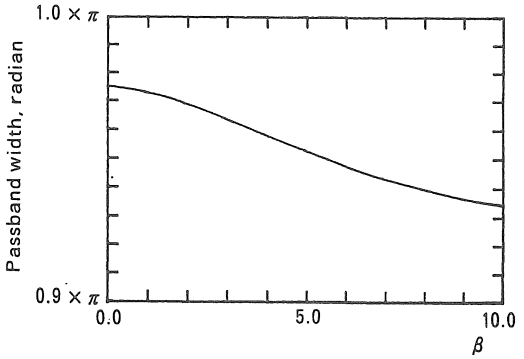

Fig.5 Bandwidth of the ST gDFT Hilbert transformer with Kaiser-weighted Nyquist vs. β (2m=8, N=8).

Following to Rabiner's discussion, the approximation error function is defined as follows.

$$E(\omega) = W(\omega)[H_d(e^{j\omega}) - H(e^{j\omega})] \tag{9}$$

where $E(\omega)$ is evaluated over both passband and eliminating band of the desired filter $W(\omega)$ as a weighting function, and $H(e^{j\omega})$ is frequency response of the optimization target.

Optimizing response in the Chebycheff meanings is estimated by the maximum absolute value of $E(\omega)$ at $M+2$ peak frequency $\{\omega_i\}$, $i = 0, 1, 2, \cdots, M+1$. These frequencies should locate in the regions $0 \le \omega \le \omega_p$ and $\omega_e \le \omega \le \pi$, here, ω_p is cut-off frequency on the passband and ω_e is cut-off frequency on the eliminating band. Where the values of the magnitude at these frequencies are given by unique value by δ.

$M+2$order simultaneous equations are derived from eq.9,

$$W(\omega_i)\left[H_d(e^{j\omega}) - h(0) - \sum_{n=1}^{M} 2h(0)\cos(\omega_i, n)\right] = -(-1)^i\delta, \qquad i = 0,\ 1,\ 2,\ \cdots,\ M+1. \tag{10}$$

There exists $M+2$unknowns for $h(n)$ and δ in these simultaneous equations. Rabiner suggested that δ is efficient to solve more than solving about $h(n)$ in concerning with peaks. In general, these peaks are given by searching around the points of dividing passband and eliminating band.

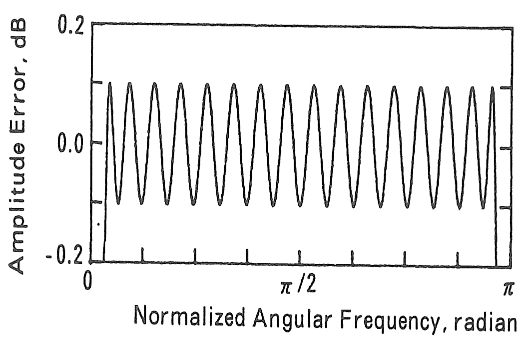

Fig.6 Amplitude response of the optimized ST gDFT Hilbert transformer, 2m=8, N=8 and δ =1.0233.

It is clearly shown in fig.6 that the amplitude response of the optimized ST gDFT Hilbert transformer is featured with equalripple within the subjective domain, where frame number $2m = 8$, frame length $N = 8$, and δ in eq.10 is set to be 0.1dB, i.e. δ =1.0233. Figure 7 shows the relationship between the amplitude error δ and passband width of the optimized ST gDFT Hilbert transformers. In similar to Kaiser weighted Nyquist, the passband width of the optimized transformer based on Remez algorithm is slightly shrinked from improving amplitude error.

4.2. Extraction of the Optimized Weighting Function

As reported previously, the unit sample response of the ST gDFT $i_g(n)$ is given as follow [4].

$$i_g(n) = \begin{cases} \dfrac{2}{N\sin(\pi n/N)} h(n), & \text{for odd } n \\ 0, & \text{for even } n \end{cases} \tag{11}$$

It is adequate that decimation function $h(n)$ described by

$$h(n) = n(n)w(n), \tag{12}$$

$n(n)$ is infinite Nyquist function and $w(n)$ is such a weighting function as Kaiser, Blackman, or the desired optimized weighting function. Substituting eq.12 into the unit sample response of eq.11, it gives,

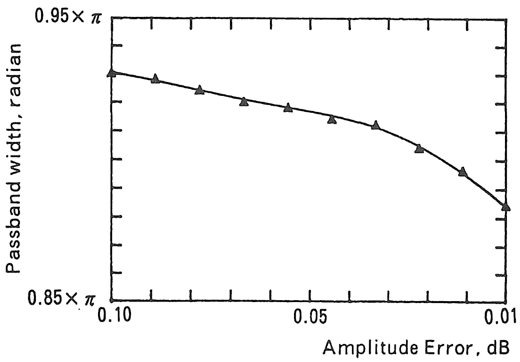

Fig.7 Passband width vs. amplitude error of the optimized ST gDFT Hilbert transformer.

$$\begin{aligned} i_g(n) &= \frac{2}{N\sin(\pi n/N)} \frac{\sin(\pi n/N)}{\pi n/N} w(n) \\ &= \frac{2}{\pi n} w(n), \qquad \text{for odd } n \end{aligned} \tag{13}$$

As well known, the term $2/\pi n$ on the right band of eq.13 means the unit sample response of the infinite Hilbert transformer. Once the unit sample response of the optimized ST gDFT Hilbert transformer, the optimized decimation function in the Chebycheff meanings is given by eq.13. That is, the optimized decimation filter, which is significant in the instantaneous spectrum analysis both in the existing ST DFT and in the ST gDFT, is defined by multiply the optimized response by the reciprocal number $\pi n/2$. Figure 8 shows the optimized weighting function by solid curve over 8 frame durations. Kaiser weighting function is also shown in the figure by dotted by curve in comparison with the optimized one.

5. CONCLUSION

The optimization of the weighting function was discussed through Remez algorithm with emphasis on reducing amplitude error both of passband and eliminating band in the meanings of Chebycheff. These optimized weighting functions ensure that such concept of the instantaneous spectrum as ST DFT, ST gDFT and etc. are put on the stage of developing

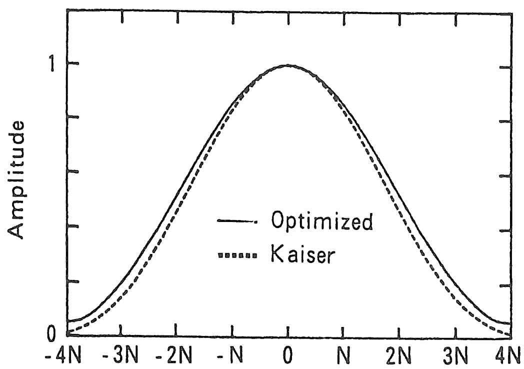

Fig.8 Amplitude response of the optimized weighted function, 2m=8.

Hilbert transformer, CODEC and MODEM optimized in the Chebycheff meanings.

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