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On the Consideration of the Generalized Short Time DFT and its Application to the Hilbert Transformer

Generalized Short Time DFTと そのヒルベルト変換への適用

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ABSTRACT *It is newly proposed in this paper based on the generalized Short Time DFT(ST gDFT) that an ideal Hilbert transformer is realized with employing phase shifting by* $-\pi/2$ *radian* $(\omega > 0)$ *and* $\pi/2$ *radian* $(\omega < 0)$. *The ST gDFT is able to adjust its sub-channel allocation by an apriori value along to the frequency axis in order to perform frequency domain Hilbert transform precisely via avoiding zero frequency crossing in the preserved sub-channel. The phase shifting function of a ST gDFT Hilbert transformer is as accurate as detecting no error with* 10^{-9} *degree order, and its amplitude is as flat as swinging within 0.01 dB over subjective domain.*

1. INTRODUCTION

Instantaneous spectrum concept is a promising solution to effectively developing key devices for economical communication systems. An exact realization of the Hilbert transformer has been previously discussed with employing new concept of instantaneous spectrum defined by ST DFT[1-4]. The Hilbert transformer used in SSB or RZ SSB modulator provides with indispensable function for eliminating one sideband from output signals to efficiently reduce occupied spectrum over radio channels[5,6].

A new class of signal processing is introduced by generalized short time DFT, in which sub-channels are arbitrary adjusted on the objective frequency domain. Another implementation of the noble Hilbert transformer is discussed with employing this ST gDFT. Restricting the ST gDFT within causality, phase shifting error of implemented Hilbert transformer is examined to be so accurate as detecting no error by micro degree order. Simultaneously, its amplitude error is shown to be less than 0.01dB.

2. PRINCIPLE OF THE HILBERT TRANSFORMER

A real signal $\hat{f}(t)$ is defined at almost all t by inverse Fourier transform from Fourier transform $F(\omega)$ of arbitrary signal $f(t)$ whose real or imaginary part is exchanged with each other. This real signal $\hat{f}(t)$ is Hilbert transform of $f(t)$. That is,

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$$\hat{f}(t) = \frac{1}{\pi}\int_0^{\infty}\{R(\omega)\sin\omega t + X(\omega)\cos\omega t\}d\omega. \quad (1)$$

Where the real part $R(\omega)$is even function given as

$$R(\omega) = \frac{1}{2}\{F(\omega) + F(-\omega)\},$$

the imaginary $X(\omega)$is odd function given as

$$X(\omega) = \frac{1}{2j}\{F(\omega) - F(-\omega)\},$$

$\hat{f}(t)$ is therefore given as follows.

$$\begin{aligned}\hat{f}(t) &= \frac{1}{\pi}\int_0^{\infty}\Big[\frac{1}{4j}\{F(\omega)+F(-\omega)\}\{e^{j\omega t} - e^{-j\omega t}\} \\ &\quad + \frac{1}{4j}\{F(\omega)-F(-\omega)\}\{e^{j\omega t} + e^{-j\omega t}\}\Big]\, d\omega \\ &= \frac{1}{2\pi}\int_0^{\infty}\{-jF(\omega)e^{j\omega t} + jF(-\omega)e^{-j\omega t}\}\ d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} -j\, sign(\omega)\, F(\omega)\, e^{j\omega t}\ d\omega.\end{aligned}$$

The Hilbert transform on the phase plane is described as

$$\hat{F}(\omega) = -j\, sign(\omega)\, F(\omega). \quad (2)$$

On the phase plane, Hilbert transform is interpreted as filtering by $-j\, sign(\omega)$. Equation 2 shows that the ideal Hilbert transform is obtained from shifting the phase by $-90°$ $(\omega > 0)$ and by $90°$ $(\omega < 0)$during signal processing based on the instantaneous spectrum of the ST gDFT.

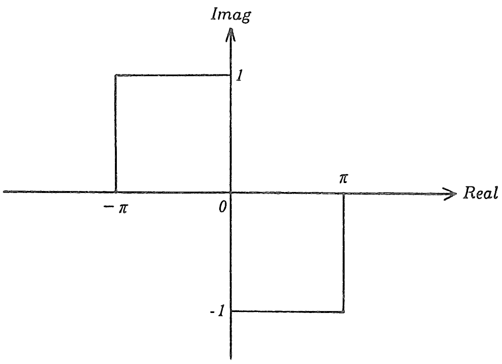

Fig.1 Frequency response of the ideal Hilbert transform in the discrete processing systems.

Figure 1 shows frequency response of the Hilbert transform as filtering in the discrete signal processing system. The unit sample response $i(n)$ of this system is given by

$$\begin{aligned} i(n) &= \frac{1}{N}\left\{\int_{-\frac{N}{2}}^{0} j e^{j\frac{2\pi}{N}\omega n} + \int_0^{\frac{N}{2}} -j e^{j\frac{2\pi}{N}\omega n}\right\} d\omega \\ &= \frac{1}{2\pi n}\{(1-e^{-j\pi n}) - (e^{j\pi n} - 1)\} \\ &= \frac{1}{\pi n}(1-\cos\pi n). \end{aligned} \quad (3)$$

This unit sample response $i(n)$ is exactly same to that of Rabiner's minimax Hilbert transformer $i_m(n)$ [7, 8].

$$i_m(n) = \begin{cases} \dfrac{2}{\pi}\dfrac{\sin^2(\frac{\pi n}{2})}{n}, & if\ n\ is\ odd \\ 0, & if\ n\ is\ even. \end{cases} \quad (4)$$

The existing DFT is impossible to be applied to the Hilbert transformer immediately because of $0th$ sub-channel existing on the frequency domain $(-\frac{\pi}{N}, \frac{\pi}{N})$ acrossing zero as shown in fig.2.

New signal processing is discussed in the following to solve this problem of $0th$ sub-channel merely by shifting channel allocation along to the frequency axis based on generalized short Time DFT, which is also newly proposing here to coincide the fringe of $0th$ sub-channel to zero.

3. GENERALIZED SHORT TIME HILBERT TRANSFORMER

3.1 Definition of the ST gDFT

We define ST gDFT and ST gIFT as follows[9].

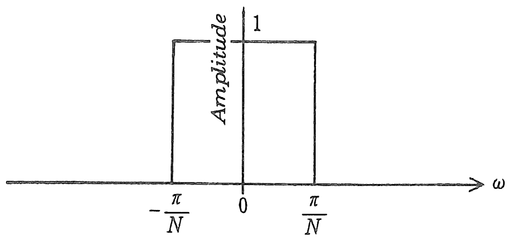

Fig.2 the 0_{th} channel allocation of the existing DFT.

$$\begin{cases} ST\,gDFT : \phi_k(n) = \sum_{r=-\infty}^{\infty} x(r)h(n-r)W_N^{-(k+\xi)r} & (5) \\ ST\,gIFT : y(n) = \frac{1}{N}\sum_{k=0}^{N-1} \phi_k(n)W_N^{n(k+\xi)} & (6) \end{cases}$$

Here, ξ is positive real number, $0 \leqq \xi < 1$, $x(r)$ is an input data at sampling time r, $h(n-r)$ is the same window functions as define in ST DFT,

$$h(p) = \begin{cases} 1, & if\, p = 0 \\ 0, & if\, p = Nu,\ u\ is\ non\ zero\ integer. \end{cases} \quad (7)$$

This satisfies physical existence and stands on causality to exist complex conjugate structure with symmetric axis at π radian among spectrum components, if ξ is $\frac{1}{2}$, as shown in fig. 3.

3.2 Unit Sample Response of ST gDFT Hilbert transfomer

Hilbert transform is exactly performed in exchanging complex components of the instantaneous spectrum on the frequency domain as previously discussed. Let real part be $R_k(n)$ and imaginary part be $X_k(n)$ of instantaneous spectrum $\phi_k(n)$. Hilbert transformed signal $\hat{y}(t)$ is given as follow.

$$\begin{aligned} \hat{y}(n) &= \frac{2}{N}\sum_{k=0}^{\frac{N}{2}-1}\Big[R_k(n)\sin\Big\{\frac{2\pi}{N}(k+\xi)n\Big\} \\ &\qquad + X_k(n)\cos\Big\{\frac{2\pi}{N}(k+\xi)n\Big\}\Big] \\ &= \frac{2}{N}\sum_{k=0}^{\frac{N}{2}-1}\Big[\sum_{r=-\infty}^{\infty} h(n-r)x(r)\cos\Big\{\frac{2\pi}{N}(k+\xi)r\Big\} \\ &\qquad \cdot \sin\Big\{\frac{2\pi}{N}(k+\xi)n\Big\} \\ &\qquad + \sum_{r=-\infty}^{\infty} h(n-r)x(r)\sin\Big\{\frac{2\pi}{N}(k+\xi)r\Big\} \\ &\qquad \cdot \cos\Big\{\frac{2\pi}{N}(k+\xi)n\Big\}\Big] \\ &= \frac{1}{N}\sum_{k=0}^{\frac{N}{2}-1}\sum_{r=-\infty}^{\infty} h(n-r)x(r)\sin\Big\{\frac{2\pi}{N}(k+\xi)(n-r)\Big\}. \quad (8) \end{aligned}$$

Restricting ξ into $\frac{1}{2}$, there exists complex conjugate relationship between $\hat{\phi}_k(n)$ and $\hat{\phi}_{N-1-k}(n)$, or between $W_N^{(k+\xi)n}$ and $W_N^{(k+\bar{\xi})n}$, eq. 8 is modified as follows.

$$\begin{aligned} \hat{y}(n) &= \frac{1}{N}\sum_{k=0}^{\frac{N}{2}-1}\Big\{\hat{\phi}_k(n)W_N^{(k+\xi)n} + \hat{\phi}_{N-1-k}(n)W_N^{(k+\bar{\xi})n}\Big\} \\ &= \frac{1}{N}\sum_{k=0}^{\frac{N}{2}-1}\Big\{\hat{\phi}_k(n)W_N^{(k+\xi)n} + \overline{\hat{\phi}_k(n)}\,\overline{W_N^{(k+\bar{\xi})n}}\Big\} \quad (9) \\ &= \frac{2}{N}\sum_{k=0}^{\frac{N}{2}-1} Real\Big\{\hat{\phi}_k(n)W_N^{(k+\xi)n}\Big\} \qquad QED. \end{aligned}$$

A Vector $\hat{\phi}_k(n)$, which is spanned by complex components mutually exchanged, is precisely coincident with the k_{th} component of Hilbert transformed instantaneous spectrum.

The unit sample response $i_g(n)$ of the ST gDFT Hilbert transformer is deduced from eq.9 by substituting unit sample $\delta(0) = 1$,

$$i_g(n) = \begin{cases} \frac{1}{N}h(n)\frac{2}{\sin\frac{n\pi}{N}}, & if\ n\ is\ odd \\ 0, & if\ n\ is\ even. \end{cases} \quad (10)$$

It is easy to understand that $i_g(n)$ converges onto unit sample response of the ideal Hilbert transform if $h(n)$ is an infinite frame number Nyquist. In fact, $h(n) = \sin(n\pi/N)/(n\pi/N)$ being substituted into eq.10, $i_g(n)$ gives ideal response as follows.

$$i_g(n) = \begin{cases} \frac{2}{n\pi}, & for\ odd\ n \\ 0, & for\ even\ n \end{cases} \quad (10)'$$

Attention must be paid on that the frame length N does not effect the unit sample response, where the ideal Hilbert transformer response is defined by that of ST gDFT Hilbert transformer as shown eq.10'. If the infinite Nyquist window is used, the ideal Hilbert transform is easy to realize but be fatal in implementation owing to output signal being de-

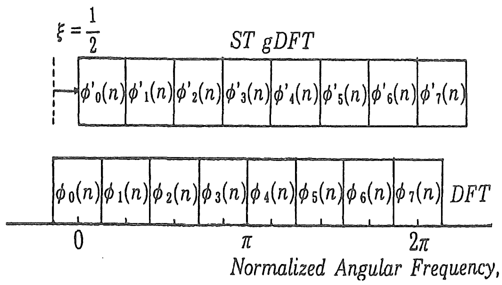

Fig.3 Comparison of sub-channel allocation between existing DFT and STgDFT, N=8.

layed by infinite duration. Fortunately, ST gDFT Hilbert transform is defined with processing input signals on the frequency domain.

Therefore, it becomes to possible in the ST gDFT to employ finite frame number $h(n)$ in order to get eq.2 by shifting the phase by $\pi/2$ radian precisely.

Consider the Kaiser smoothing function to truncate infinite Nyquist by finite frame number[10],

$$h(n)=N(n)K(\beta,\ n) \tag{11}$$

here, $N(n)$ is infinite Nyquist

$$N(n)=\frac{\sin\frac{n\pi}{N}}{\frac{n\pi}{N}}, \tag{12}$$

and $K(\beta,\ n)$ is truncating function

$$K(\beta,\ n)=\frac{I_0\left(\beta\sqrt{1-\frac{n^2}{m^2N^2}}\right)}{I_0(\beta)}, \tag{13}$$

$$-mN \leqq n \leqq mN.$$

Where, $I_0(*)$ is the modified 0_{th} order first kind Bessel function, β is arbitrary value to adjust width and energy of the mainlobe.

It will be shown in the next session that the truncated window $h(*)$ is approximately adjusted to coincide with the infinite Nyquist with selecting β by apriori values. Function $N(n)K(\beta,\ n)$ is especially called by Nyquist-Kaiser in the following.

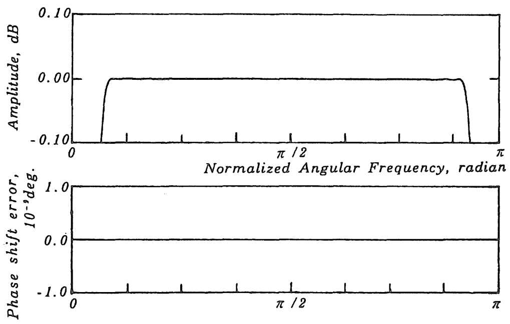

Fig.4 Frequency response of amplitude and phase shift error of the STgDFT Hilbert transformaer, 2m=8 and β =9.0.

4. COMPUTER SIMULATIONS AND RESULTS

The amplitude frequency response is shown in fig.4 for unit sample response of the ST gDFT Hilbert transformer, where the frame number $2m$ is set to be 8 and β of Nyquist-Kaiser is taken as 9. As shown in flatness over subjective domain, the optimized approximation is easily given by adjusting the value of β. It is also mentioned that there exists any phase shift error in accuracy of micro degree order.

Figure 5 (a) shows the amplitude response of the ST gDFT Hilbert transformer as the frame number $2m$ being taken as a parameter when the Nyquist-Kaiser window length $2mN$ is set to be 64 and β is 6. Under the same conditions in the above, figure 5 (b) shows the amplitude response as the frame length N being taken as a parameter. As shown in these figures, the Hilbert transformer is low in sensitivity to cause no changes in amplitude characteristics if the parameter $2m$ or N changes.

Even if it gives good characteristics when infinite Nyquist being employed, it is not practicable because of being large in processing delay. It is easy to understand that only the single function of the Hilbert transformer is also realized with transversal filters when the unit sample response $i_g(n)$ of the ST gDFT Hilbert

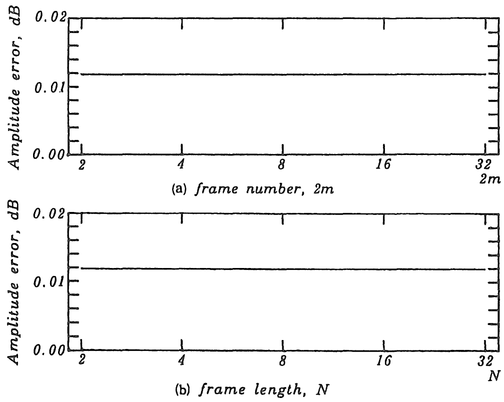

Fig.5 amplitude error vs. frame number 2m(a), and amplitude error vs. frame length N(b) of the STgDFT Hilbert transformer, 2mN=64 and β=6.0.

transformer is exactly given. This means the delay of ST gDFT Hilbert transformer is given by $mN\tau$. Here, τ is reciprocal number of sampling frequency.

Figure 6 shows the amplitude response as delay mN being taken as a parameter. The amplitude error of the ST gDFT Hilbert transformer is shown to be practicable from the value observed in fig.6 to be below 0.01dB when its processing delay is restricted within 16 msec. in the case of 8 kHz sampling as standard in communication signal processing.

5. CONCLUSION

The generalized short time DFT (ST gDFT) was successfully shown to be deduced from adjusting allocation of sub-channels with emphasis on realization of the noble Hilbert transformer which is inevitable in improving frequency utility efficient of communication systems. The unit sample response of the ST gDFT Hilbert transformer which employs the infinite Nyquist window is precisely coincide with that of ideal Hilbert transformer with fatal demerit of astronomical delay. However, the ST gDFT Hilbert transformer is executed with signal processing on the phase plane through instantaneous spectrum analysis and synthesis based on the ST gDFT to avoid this fatal demerit and to be able to get exactly Hilbert transformed signal within practicable delay, $mN\tau$.

The Kaiser function introduced into the ST gDFT is also able to speed up the signal processing by truncating Nyquist function without increasing both phase shifting and amplitude errors. The ST gDFT Hilbert transformer is shown to be released from the restrict conditions of frame number $2m$ and frame length N and shown to be depend on only the product $2m \times N$ which is proportional to the delay amount. The frequency responses are verified through computer simulations to be so accurate as less than micro degree order in phase shifting and less than 0.01dB in amplitude error.

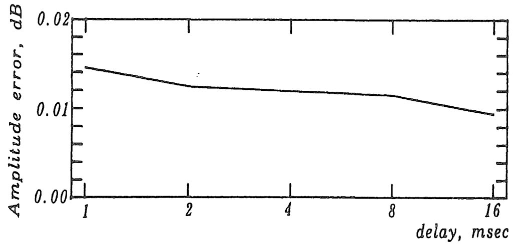

Fig.6 Amplitude error vs. delay mN of the STgDFT Hilbert transformer, β=6.0 and sampling rate is 8kHz.

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