Plastic Torsion Tests with Mild Steel Shafts

No.9, Shafts with Keyseat (Part-1)

軟鋼軸の塑性ねじりに関する実験

第９報　Keyseat を有する軸（その１）

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Abstract In order to clarify the effects of notches on yielding, strain figures developed in notched shafts are observed in details during the plastic stage of elastic-plastic torsion. The test pieces used are 0.36% C carbon steel shafts having a keyseat taken in a broad sense. The torsional moment- deflection curves are obtained throughout the elastic-plastic stage of torsion and the values of torsional moment are obtained for each notch. Constraint factors for each notch are given and the influence of the shape of notch on the factors is investigated. Comparison is made with the theoretical results obtained previously in the case of yield condition of constant maximum shearing stress.

1. Introduction

The problem of searching for yield point load is an important subject to make clear the material's plastic deformation mechanism and to gain a fundamental data on plastic design. Therefore it's a practically important study assignment as a fundamental problem to search for yield point load of a notch material with yield torson[1, 2]. Especially when we take the character of an actual material into consideration we must be based on the experimental methods. Previously I made plastic torsion tests with mild steel shafts having circular, square and rectangular[3] cross sections of the same cross section area, and mild steel shafts having U- notched cicumferential groove[4, 5], rectangulr-notched circumferential groove[6] and End Milled Keyseat[7], observed the relation between the development of plastic region and torsional moment in detail, made plastic deformation mechanism clear and measured yield point torsional moment.

In this study, I made a elastic-plastic torsion test with mild solid shaft having infinite long keyseat along the shaft. Here I selected nine types of notched shape which had fixed shaft's diameter (D) and radius of curvature at the bottom corner of the keyseat (r) and different breadth (b) and depth (t) of the keyseat and drew a torsional moment-angle of torsion figure at each stage of elastic-plastic torsion. I threw light on the relation

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between the development of prastic region which appeared on the minimum cross section which includes keyseat and torsional moment-angle of torsion at each stage of plastic torsion. I showed how the difference of the shape of keyseat effected on the development of plastic region. And I measured approximate yield point torsional moment and especially in the case of the test piece without keyseat, comparing with the theoretical figure calculated as a perfectly plastic material [8, 9]. I calculated the constraint factors which indicated the ratio of restriction of elastic region that influenced the development of plastic region corresponding to the difference of shape of keyseat.

So far, A.NADAI [10] and Л.М.КАЧАНОВ [11] have made similar research with strain figures but I've never seen a study which dealt with keyseat. In this experiment using existent materials manufactured and sold in this country, I observed the plastic region which developed on the keyseat and in the shafts minutely.

2. Experimental Method

2.1 Test piece

The material of test pieces is S35C drawn steel which has been given heat treatment (kept at the temperature of 880℃ for 135minutes, then air-cooled; kept at the temperature of 700℃ for 60minutes, then air-cooled). Table 1,2 show the chemistrical compositions and the mechanical properties of the material.The shafts in this test have the same diameter and nine types of keyseat, differing in breadth (b) and depth (t), yet each bottom corner has the same radius of curvature (r). Every test piece is deliberately completed, and fine pieces are selected through inspection of radius of curvature by projector. Fig. 1 shows form of the test piece.

The size of every part is as follows

D = 20.00mm , r = 0.00mm
b = 3.00 , 5.00 , 7.00mm
t = 2.00 , 4.00 , 6.00mm
guage length (l) = 50.00mm
length of test piece (L)=300±1mm

Table 1 Chemical Compositions (%)

C	Si	Mn	P	S	Cu	Ni	Cr	Fe
0.36	0.23	0.64	0.014	0.032	0.19	0.06	0.12	remainder

Table 2 Mechanical Properties

Youngs Modulus E GPa	Modulus of Rigidity G GPa	Yield Point σ_s MPa	Tensile Strength σ_B MPa	Elong. ϕ %	Reduction in Area ψ %
208	80.9	396	594	34.4	65.0

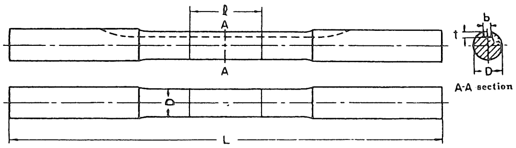


Fig.1 Test piece

2.2 Experimental Method

The experiment is made using pendulum torsional testing machine, and angle of torsion is measured by optical lever. Loading is through hand operation, and at each loading stage, attention is paid to keep constant loading speed. Till yield region gets wide, in every loading stage same quantity of load is added and angle of torsion is measured. As yield region gets wide to a certain extent, loading becomes unstable because of strong local slip. In this case, after loading gets stable enough, angle of torsion is measured and more load is added. Loading is ended when strain hardening becomes obvious. Then, a part of the test piece is cut off and strain figure is detected after etching [12].

3. The result and consideration

Figures 2 to 4 show the results of the experiment of nine types of test pieces with different shapes of keyseat. These are the torsional moment (T)- angle of torsion (θ) figures of every value of width (b), depth (t) taken as a parameter. Figure 5 is a T-θ figure of a test piece without keyseat. Figures 6 to 8 show the figures of the development of plastic region in the minimum cross section of test pieces. The numbers in the T-θ figures correspond to the photographs of strain figures with the same numbers, and these photographs were taken after loading was finished and the pieces were etched. And ④ in Figure 5 is the point when the strain figure is photographed which shows the approximate yield point torsional moment of a test piece without keyseat, and for details the references[13], Figure 2" should be refered to.

3.1 T-θ figure and Development of plastic region

In the case of test piece with keyseat it is the place[14] in which stress concentrates by elastic torsion and shearing stress become maximum to reach the plastic region at first, namely, point (m) of figure 10, the both corners of bottom of groove. At figures 2 to 4 ① shows the early condition of plastic region which develops from stress concentrated point (m). In this stage plastic deformation is very slight as the figure displays. After T increases and yield makes progress to a certain degree, T-θ figure goes away from straight part showing elastic deformation. In this stage plastic deformation is confined to the almost same degree with elastic deformation. Further adding torsional moment (T), suddenly, T-θ figure starts to curve. Plastic region appearing at the bottom of groove, as widening and increasing in number, makes progress to the center, on the other hand also from the part of circumference which has been elastic so far, plastic region appears and develops perpendicularly to outline (②). Still more T increases, then T-θ figure curves harder abruptly, slip occurs frequently at the adjoining part. And plastic region becomes wedge, as widening and increasing in number, and makes progress inward (③).T-θ figure becomes at an almost level only with slight inclination. At ④ T-θ figure arrives, then bottom of groove and neighbourhood of circumference become plastic region almost completely, and there stress value becomes equal with yield stress value τ_s. Soon hardening region is developed first from these complete yield part gradually, on the other hand yield region of the inside extends toward the center and the inclination of T-θ figure increases. Still more and more increasing load, as elastic region, only a little part enclosing the discontinuous line[1] [2] of stress is left because of wedge-shaped development of plastic region, and then outline and it's neighbourhood become the region giving rise to strain hardening (⑤). Besides it is observed that linear plastic region appears along the axis on the curved surface part of test piece. Figure 6 ~ 9 shows the figures of development of plastic region after excluding the load (nearby 12.5 deg/50mm : ⑤) of final stage from each test piece, and we can understand that in the central part between bottom of groove and circular outline on the cross section there is the center of torsion.

3.2 Yield point torsional moment

We can understand that almost all the area of the minimum cross section of keyseat (which involves profile line surface) becomes plastic region[13] by the strain figures which show the condition of development of plastic region at figures 6 to 8. On the other

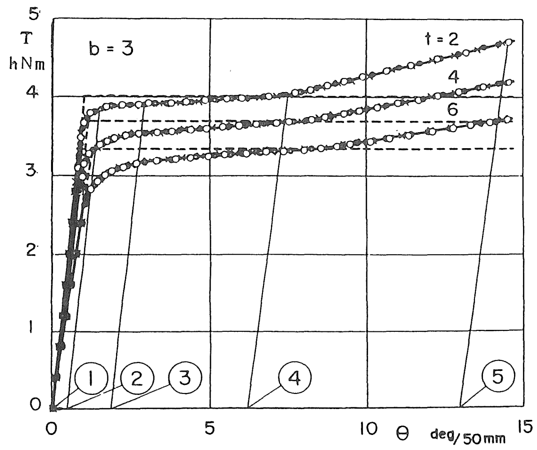


Fig. 2

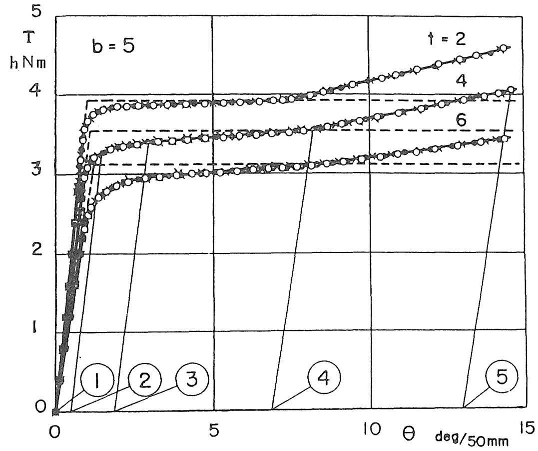


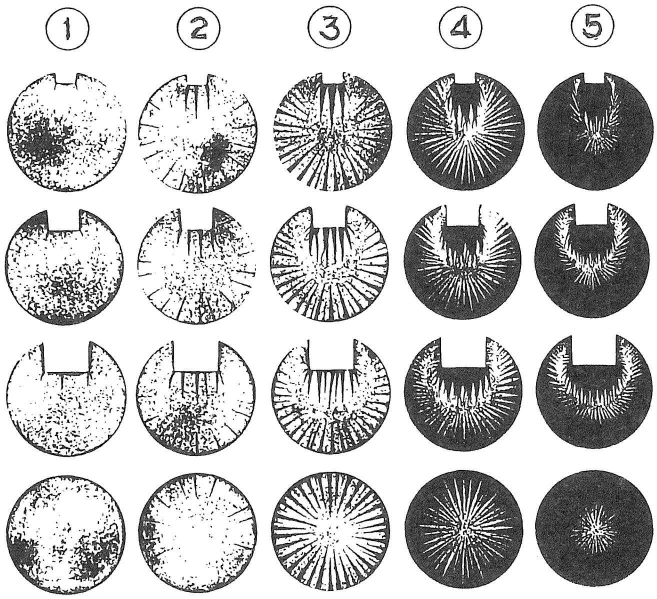
Fig. 3

Fig. 6

Fig. 7

Fig. 8

Fig. 9



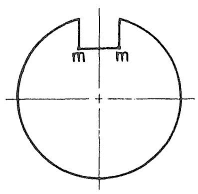


Fig. 10

Stress concentration point

Fig.2~9 Torque-twist diagram of a mild steel showing flow or strain figures.

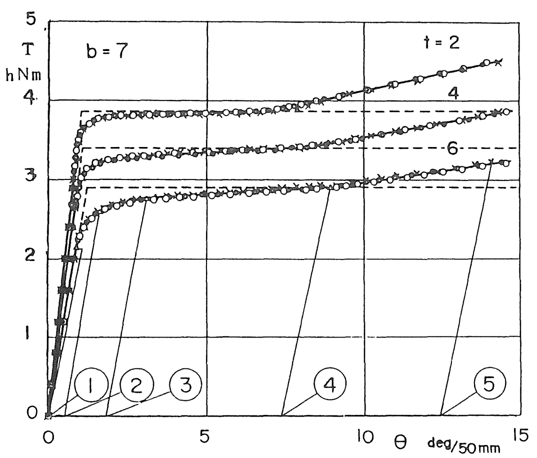


Fig. 4

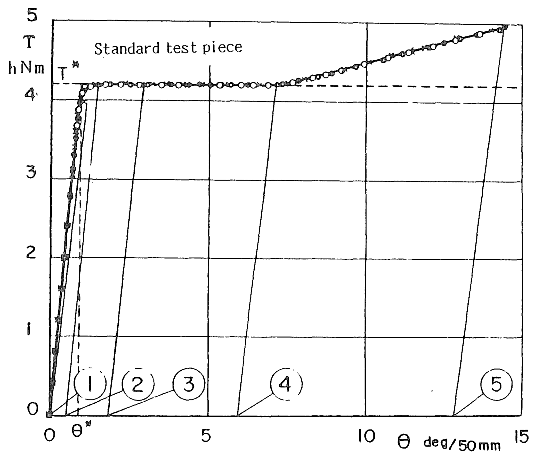


Fig. 5

hand the experimental effects of figures 2 to 4 show that bigger torsional moment is needed to grow the transformation beyond the point ④,that is to say, the position of point ④ is found easily because in the area beyond point ④, it is recognized that the inclination against axis θ of T-θ figure increases suddenly. In this way,what is just before the starting of strain hardening, or torsional moment corresponding to point ④ approximately gives yield point torsional moment T_0 concerning perfectly plastic material. Namely we can consider it , in which the horizontal line passing point ④, shown as broken lines in figures 2 to 4 and the extended line from elastic part are connected, to show the case that shaft material is made of perfectly plastic material. Table 3 shows yield point torsional moment T_0 of test piece with keyseat using T-θ figure and table 4 shows, T^*, yield point torsional moment of test piece without keyseat and maximum angle of torsion, θ^*, supposed to be twisted elastically by T^*. As for the theoretical value [2] calculated by supposing pure plastic stress condition appearing corresponding to infinite and relative angle of torsion, yield point torsional moment T_{th}^* and angle of torsional θ_{th}^* supposed to be twisted elastically are each given by the expression (1).

$$T_{th}^* = \frac{2}{3}\pi\kappa\left(\frac{D}{2}\right)^3, \quad \theta_{th}^* = \frac{32T_{th}^*\ell}{\pi D^4 G} \qquad \cdots\cdots (1)$$

In the expression (1) κ is a quorum decided corresponding to plastic condition. The result calculated by putting the value, $\kappa = \tau_s = \sigma_s/2$ being based on the condition of fixed maximum shear stress, into expression is shown in the table 4, and it accords with experimental value very well. Therefore we can also regard the case of other test piece with keyseat as what is reliable.

Table 3 Yield point tosional moment

T_0 Nm

t \ b	3	5	7
2	400.9	392.0	385.3
4	368.8	353.8	340.1
6	334.1	311.7	290.5

Table 4 Standard test piece ; T^*, Θ^* (test piece without keyseat)

D mm	T^* Nm	T_{th} Nm	T_{th}/T^*	Θ^* deg/50mm	Θ_{th} deg/50mm
20.00	420.0	414.7	0.987	0.925	0.936

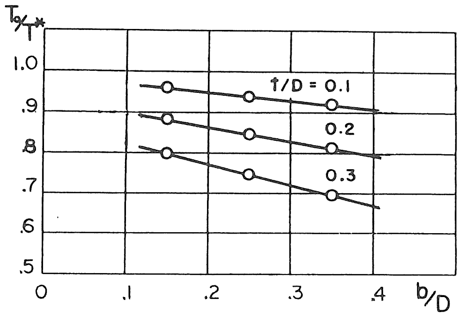


Fig. 11 Constraint factor

3.3 Constraint factors

I show yield point torsional moments of each piece, T_0, T^*, read from T-θ figures. And I calculate the ratio of restriction of elastic region that influences the development of plastic region corresponding to difference of shape of keyseat, that is to say, the constraint factors T_0/T^*, and then show the relation of T_0/T^* and b/D which is a ratio of breadth of keyseat (b) and diameter of shafts (D) at figure 11. We can read it from table 3 and figure 11 that as the breadth of keyseat (b) becomes wider, as the depth of grooves (t) becomes deeper, yield point torsional moment T_0 decreases, that is to say, as the resistance against torsion decreases gradually, and conversely as the values of (b)·(t) decrease , the value of T_0 gets closer to T^* by degrees.

4. Conclusions

I clarified the relation between the development of plastic region and the torsional moment by making elastic-plastic torsional experiment on nine types of mild steel shafts which have keyseat, drawing T-θ figure, and detecting strain figure of the minimum cross section. I calculated the approximate yield point torsional moment on actual shaft materials, compared with the theory that supposes the pure plastic stress condition especially in the case of test pieces without keyseat, and showed that the maximum shear stress corresponds well with the theoretical value calculated under a certain plastic condition. I calculated the constraint factors which indicates restriction of elastic region on the development of plastic region.

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