The Effect of Frame Truncation Error Reduced by the Short Time DFT

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Short Time DFT におけるフレーム端効果の抑圧

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The effect of frame truncation error is discussed with emphasis on difference between existing DFT and short time DFT. Existing DFTs suffer from fluctuations on spectrum analysis results caused by limitting into a frame.

In this paper, short time DFT is discussed to be free from frame truncation error and to introduce various profit into spectrum analyzers.

1. Introduction

Existing discrete Fourie transform (DFT) is widely used as digital spectrum analyzers, namely FFT analyzers. However, it is pointed out that a frequency response obtained by existing DFT suffers from significant fluctuation due to the frame setting on the sampling data stream.

This error, called by "the frame truncation error", degradates both the SNR and frequency resolution of existing FFT spectrum analyzers. Such window functions as Blackman and Hanning are usually used to suppress the frame truncation error. Unfortunately, these windows are not enough to suppress the frame truncation error in DFT analysis.

In the paper, it is examined to apply the short time DFT to a spectrum analyzer free from the frame truncation error(1).

2. Difference between DFT and short time DFT

The spectrum Φ given by DFT is as follows,

$$\Phi = [\phi_0 \ \phi_1 \cdots \phi_{N-1}]^T \tag{1}$$

Here,

$$\phi_k = \sum_{i=0}^{N-1} h_i \, x_i \, W_N^{-ik}, \tag{2}$$

$W_N^{-ik} = \exp\{-j(2\pi ik/N)\}$,

k is integer and $0 \leqq k < N$.

Where,

x_i are input sampled data,

h_i is window function,

N is number of frame samples.

The spectrum Φ given by exisitng DFT is seemed to be average spectrum over the frame duration, because Φ is equally corresponding to all of the data included in the frame.

On the other hand, the short time DFT gives the instantaneous spectrum, $\Phi(n)$, as follows,

$$\Phi(n) = [\phi_0(n)\phi_1(n)\cdots\phi_{N-1}(n)]^T \tag{3}$$

Here,

$$\phi_k(n) = \sum_{r=-\infty}^{\infty} x(r)h(n-r)\,W_N^{-rk}. \tag{4}$$

Where,

$x(r)$ are input samples data,

$h(*)$ is window function,

$$h(mN) = \begin{cases} 1, \text{ if } m \text{ is } 0. \\ 0, \text{ if } m \text{ is non-zero integer.} \end{cases} \tag{5}$$

Attention must be paid on that the parameter r of Eq. (4) varies from $-\infty$ to ∞. That is, Eq. (4) requires infinite product operations to get $x(r)$ at every time r. It takes astronomical time and infinite time-delay that Eq. (4) is carried out literally.

The truncated window, $h(*)$, with finite value r, is, therefore, parctically employed in short time DFT. However, the truncation of $h(*)$, under than given limitation causes such effects as getting worse SNR, losing sharpness of cut-off characteristics, and etc. The problem of finding the limitation for truncation will be discussed in another article(2) and beyond this discussion.

There exists significant difference between the average spectrum given by DFT and the instantaneous spectrum given by short time DFT. The short time DFT is able to give instantaneous spectrum $\Phi(n)$ at every sampling time and make free from the fluctuation according to how setting the frame duration on the sampling data stream. This benefit is produced from the sacrifice of paying the more operation than that of DFT requires. However, the sacrifice of paying the great deal of operation is compensated with remarkable advance of semiconductor in recent days. For example, 100 MIPS requires such super computer as CRAY, in general purposes, but it is easily offered by 10 chips of such digital signal processing processors as Texas TMS, Fujitsu MB, NEC μPD, and etc.

According to these remarkable advances in semiconductor technology, short time DFT is put on the practical stage of applying to spectrum analysis.

3. Experiment of the Frame Truncation Error Reduction

Experimentations are carried out under following assumptions to confirm the frame truncation error reduction via the short time DFT.

[Assumption 1] All of the signals have 0 to $7\pi/8$ radian phase delay at the front-end of the frame as shown in Fig. 1. For example, Fig. 1 shows a frame allocation of zero phase delay by broken lines. It also shows a frame allocation of $\pi/4$, or $\pi/2$, by chained or solid lines, respectively. Here, this phase delay is defined to allocate the frame position for every signals without any loss of generality. When the input signal is set to be given frequency, the iteration within the frame increase as making the frame duration longer. In this case, it is able to think that the iteration of input signals changes within the various frame durations. Therefore, we defined this iteration number which is count under restrictions mentioned above merely by "iteration number" without any confusion in discussion.

[Assumption 2] All of the input signals are tonal, because both DFT and short time DFT are linear.

[Assumption 3] The window function is Hanning as given by Eq. (6) in DFT, or truncated Nyquist with eight frame number as given by Eq. (7) in short time DFT, respectively.

$$h_k = 0.5 - 0.5\cos(2\pi k/N). \quad (6)$$

$$h(n) = \sin(n\pi/N)/(n\pi/N). \quad (7)$$

For the case of discussion, the experimentations are also demonstrated under following conditions: the sampling rate is set to be 8kHz, the sample nember, N, is given by power of 2, to cover range of 8 ($=2^3$) to 8,192 ($=2^{13}$), the total input signal takes one value among 0.95, 1.00, and 1.05 kHz.

As well known, the degradation of frequency resolution occurs by exisiting DFT in the case that input signals have discontinuity as shown in Fig. 2. The short time DFT is able to sufficiently suppress the degradations under the same conditions as shown in the same figure. The ability of suppressing degradation of frequency resolution almost always takes constant values for variation of the phase delay.

Figure 3 shows purity as taking the iteration number as parameters. Here, purity is defined by the amount of difference between input signal power and neighboring power of the close frequency index. Purity both of DFT and short time DFT slightly undulate in increasing the iteration number. However, short time DFT's purity is superior to that of existing DFT by more than 20

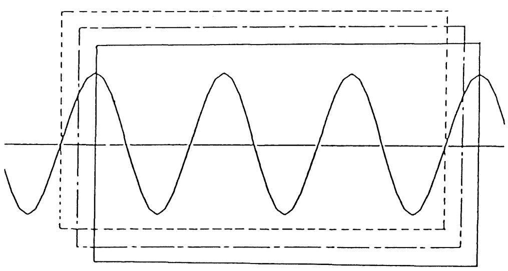

Fig 1 Relationship of the phase delay between input signal zero crossing and front-end of the frame.

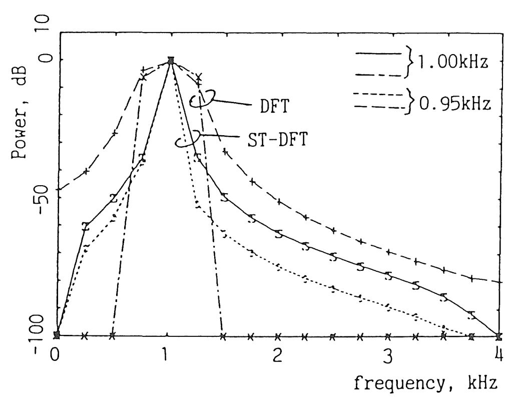

Fig 2 Power spectrum responses analyzed existing DFT and the short time DFT, $N=32$.

dB.

4. Conclusion

The frame truncation error has been discused with emphasis on difference between existing DFT and short time DFT.

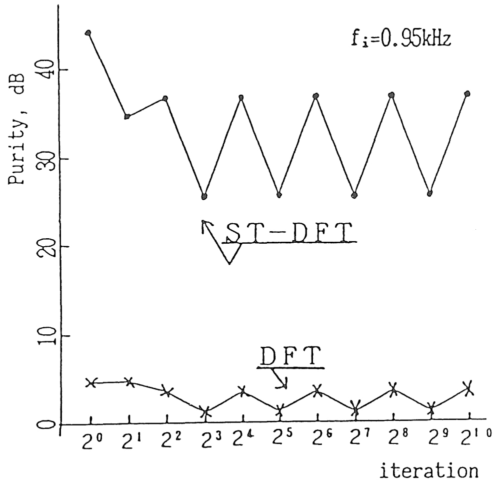

Fig 3 Comparison with purity characteristic over iterations of DFT and short time DFT, $f_i = 0.95$kHz.

Spectrum analyzer based on the existing DFT can show their high performance in the limitations of and restrict periodic at front and tail-end of the DFT frame and of that the signals are precisely sub-harmonics of the DFT sampling frequency.

On the other hand, spectrum analyzers which employ the short time DFT can show their high performance for arbitrary input signals without any restrictions of both zero crossing and subharmonics of the sampling rate. The application of short time DFT to spectrum analyzers yields efficient the frame truncation error reduction to negligible small and brings into the great deal of processing operations which will be easily compensated with progress in semiconductor technology in these days.

References

(1) T. Saiki and M. Kishi : "Reduction the Effect of Frame Truncation Error Using the Short Time DFT", Trans. IEICE, E71, 5, pp. 472-474 (May 1988).

(2) K. Mizuno and M. Kishi : "An Optimization of the Decimation Filter Used in Short Time DFT Hilbert Transfomers", Trans. IEICE, E71, 5, pp. 469-471 (May 1988).

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