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Existence of Hilbert Transformer which Allows Only Instantaneous Spectrum Processing and its Circuitry Scheme

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瞬時スペクトラム操作のみを許すHilbert変換機の存在とその回路構造

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Hilbert transform has been previouly studied in many fields, as it plays significant and indispensable rolles in Single Side Band (ab. in SSB) modulation to save occupied frequency bandwidth. It is unfortunate that the Hilbert transform has been realized to possess phase shifting characteristic by $\pi/2$ radians from the point of view for circuitry approximation.

In this paper, an ideal and exact realization of the Hilbert transform is shown clearly by adopting instantaneous spectrum analysis and synthesis, namely Short Time DFT (ab. in ST-DFT). Where the instantaneous spectrum is given, the Hilbert transform on the frequency domain is precisely carried out via merely exchanging both original real or imaginary part of the spectrum into mapped imaginary or real part, respectively. The Hilbert transform is, therefore, performed as synthesizing the output signal correspondence with the transformed instantaneous spectrum via inverse ST-DFT (ab. in ST-IFT).

A circuitry configuration and its characteristic of the short time DFT Hilbert transform are demonstrated to be ideal through unit sample responce.

1. INTRODUCTION

Capability of preventing single side band modulation from increasing frequency bandwidth over radio channels puts Hilbert transformers onto a significant stage of communication system.

In this paper, a nobel Hilbert transformer is proposed by adopting instantantauous spectrum analysis named by Short Time DFT and the transformer configuration is also discussed.

2. SHORT TIME DFT AND ITS CHARACTERISTICS

Consider first how the ST-DFT gives the instantaneous spectrum and what characteristics it possesses. Let instantaneous spectrum at sampling time n be described by Φ (n) as follows,

$$\Phi\ (n) = \{\phi_0(n)\ \phi_1(n)\ \phi_2(n)\ .\ .\ .\ \phi_{N-1}\ (n)\}^T. \tag{1}$$

Where, ϕ_k (n) is a spectrum component at frequency index k of Φ (n) given as,

$$\phi_k\ (n) = \sum_{r=-\infty}^{\infty} x\ (r)\ h\ (n-r)\ W_N^{-rk}, \tag{2}$$

integer k is $0 \leqq k < N$.

Here, x (r) is an input data at sampling time r. Every W_N^{-s} is the same operator defined for the existing DFT.

Existence of ST-DFT requires that output signal at time n, y (n), is precisely produced from the instantenuous spectrum Φ (n) via the short time inverse DFT (ab. in ST-IFT) as follows,

$$N \cdot y\ (n) = \sum_{k=0}^{N-1} \phi_k(n)\ W_N^{nk}. \tag{3}$$

Here, every W_N^k is the same operator defined for existing inverse DFT (ab. in IFT), $W_N^k = \exp\{j(2\pi k/N)\}$.

Since the ST-IFT is denoted by eq. 3, the window function, h (*) of eq. 2, is necessary to hold the condition, y (n)=x (n). Substituting the eq. 2 into eq. 3 and exchanging the summation order for variables k and r, it gives eq. 4 as follows,

$$N\cdot y(n)=\Sigma_{k=0}^{N-1}\{\Sigma_{r=-\infty}^{\infty}x(r)h(n-r)W_N^{-rk}\}W_N^{nk}$$

$$=\Sigma_{r=-\infty}^{\infty}x(r)\,h(n-r)\{\Sigma_{k=0}^{N-1}W_N^{(n-r)k}\}. \quad (4)$$

The summation for variable k takes non zero value N, only if n−r=2Nq. Here, q is an integer. This condition gives the window function a restriction as follows,

$$h(p)=\begin{cases}1, & p=0\\ 0, & p=2Nu,\ u \text{ is non zero integer.}\end{cases} \quad (5)$$

A truncated Nyquist window function with 2m frame number and N frame length, h (p),

$$h(p)=\sin(p\pi/N)/(p\pi/N), \quad -mN\leqq p\leqq mN, \quad (6)$$

satisfies eq. 5. Hereafter, it will be employed as windor function of ST-DFT.

3. PRINCIPLE OF ST-DFT HILBERT TRANSFORMER

The Hilbert transform on the frequency domain is performed merely by exchanging the real part for imaginary part, that is, it is performed merely by multiplying complex unit −j both real and imaginary components. Here, $j=\sqrt{-1}$. Both the ST-DFT operation and phase shifting by $\pi/2$ radian being combined into one operator, the ST-DFT Hilbert transform operator $\hat{W}_N^{-rk}$ is given as follows,

$$\hat{W}_N^{-rk}=\begin{cases}\exp\{-j(2\pi rk/N+\pi/2)\}, & \text{if } 0<k<N/2\\ 0, & \text{if } k=0,\ N/2\\ \exp[j\{2\pi r(N-k)/N+\pi/2\}] = \exp\{-j(2\pi rk/N-\pi/2)\}, & \text{if } N/2<k<N.\end{cases} \quad (7)$$

PROOF: Existence of ST-DFT Hilbert Transformer

The first row on the right hand of eq. 7 gives the characteristic which features of phase shifting by $\pi/2$ radian. This also clearly shows that ST-DFT Hilbert transformer is free from amplitude distortion, because the operators consist of single complex function only with pure imaginary variables. That is,

$$\hat{W}_N^{-rk}=\exp\{-j(2\pi rk/N+\pi/2)\}$$
$$=-j\cdot\exp(-j2\pi rk/N)$$
$$=-jW_N^{-rk}.$$

That is,
$$|\hat{W}_N^{-rk}|=|W_N^{-rk}|,$$

here, | * | means the absolute value of complex variable *.

Both the second and third rows of eq. 7 promise that the output signal is able to exist as a physical variable. The output signal $\hat{y}$ (n) is given by substituting the Hilbert transformed spectrum $\hat{\phi}_k(n)$ into $\phi_k(n)$ of eq. 3. That is,

$$N\cdot\hat{y}(n)=\Sigma_{k=0}^{N-1}\hat{\phi}_k(n)\,W_N^{nk}$$
$$=\hat{\phi}_0(n)\,W_N^{0}+\hat{\phi}_{N/2}(n)\,W_N^{nN/2}$$
$$+\Sigma_{k=1}^{N/2-1}\{\hat{\phi}_k(n)\,W_N^{nk}+\hat{\phi}_{N-k}(n)\,W_N^{n(N-k)}\} \quad (8)$$

In the first and second terms on the right hand of eq. 8, IFT operator W_N^0 or $W_N^{nN/2}$ takes a pure real number 1 or 1 (−1), respectively. If the spectrum $\hat{\phi}_0(n)$ or $\hat{\phi}_{N/2}(n)$ is non zero, the output $\hat{y}$ (n) is diversified into a complex number. The restriction of $\hat{y}$ (n) being a real number also requires that the third term of eq. 8 is also pure real.

Since $W_N^{n(N-k)}$ is described by W_N as follows,

$$W_N^{n(N-k)}=\exp\{-j2\pi n(N-k)/N)\}$$
$$=\exp(j2\pi nk/N)=\bar{W}_N^{nk},$$

every component of the third term in eq. 8 takes a pure real number, iff $\hat{\phi}_{N-k}(n)$ is complex conjugate with $\hat{\phi}_k(n)$. In practice, there exists following relation between $\hat{\phi}_k(n)$ and $\hat{\phi}_{N-k}(n)$,

$$\hat{\phi}_{N-k}(n)=\Sigma_{r=-\infty}^{\infty}x(r)\,h(n-r)\,\hat{W}_{N-k}^{-r(N-k)}$$
$$=\Sigma_{r=-\infty}^{\infty}x(r)\,h(n-r)\exp\{j(2\pi rk/N+\pi/2)\}$$
$$=\text{conj}[\Sigma_{r=-\infty}^{\infty}x(r)\,h(n-r)\exp\{-j(2\pi rk/N$$
$$+\pi/2)\}]=\bar{\hat{\phi}}_k(n).$$

Using the short time DFT Hilbert transform operators define by eq. 7, the output signal of the short time DFT Hilbert transformer is given to be real as follows,

$$
\begin{aligned}
N\cdot\hat{y}(n) &= \Sigma_{k=1}^{N/2-1}\{\hat{\phi}_k(n)\ W_N^{nk}+\hat{\phi}_{N-k}(n)\ W_N^{n(N-k)}\} \\
&= \Sigma_{k=1}^{N/2-1}\{\hat{\phi}_k(n)\ W_N^{nk}+\bar{\hat{\phi}}_k(n)\ \bar{W}_N^{nk}\} \\
&= \Sigma_{k=1}^{N/2-1}2\mathrm{Real}\{\hat{\phi}_k(n)\ W_N^{nk}\}, \qquad \text{QED.}
\end{aligned}
$$

4. CIRCUITRY CONFIGURATION AND ITS UNIT SAMPLE RESPONSE

Figure 1 shows a primitive block diagram of ST-DFT Hilbert transformers. ST-DFT Hilbert transformers are categorized into three functional blocks. The first functional block plays a role of ST-DFT analayzers and consists of N/2-1 channel modules in which every component $\phi_k(n)$ of the instantanuous spectrum is yieled, here k=1, 2, . . . , N/2-1. The second block acts as a Hilbert transformer on the frequency domain. This block is dominant in function, however, its circuitry configuration is so simple as it only posesses two crossing wires as shown in fig. 1. The last is ST-IFT to produce the time domain signal. In practice, ST-DFT Hilbert transformers are also able to consist of employing fast Fourier transform (FFT) instead of DFT to save signal processing power. The detailed configuration of short time FFT Hilbert transformers will be discussed in other materials in the near future.

The ST-DFT Hilbert transformer is substantiated through unit sample response, which is corresponding with unit impulse response for continuous signals. Figure 2 (a) shows unit sample response, and fig. 2 (b) shows spectrum characteristics. The experimentations were held under the following condition: the window function frame number m and frame length N are set to be 8 and 32, respectively, the spectrum is analayzed through 1,024 samples with centering to the zero time of the unit sample response.

As shown in fig. 2 (a), unit sample response is almost equivalent to that of ideal Hilbert transformers. This ideal characteristic is also observed upon the frequency domain as shown in fig. 2 (b), the power spectrum illustrated in fig. 2 (b) shows flatness over all frequency band except the neighboring domains of the edges of the three elimination bands at 0, π, and 2π radians with $2\pi/N$ radian bandwidth.

5. CONCLUSION

An exact realization of the Hilbert transformers has been succesfully discussed with employing instantanuous spectrum analysis instead of all pass filter approximations. This exact

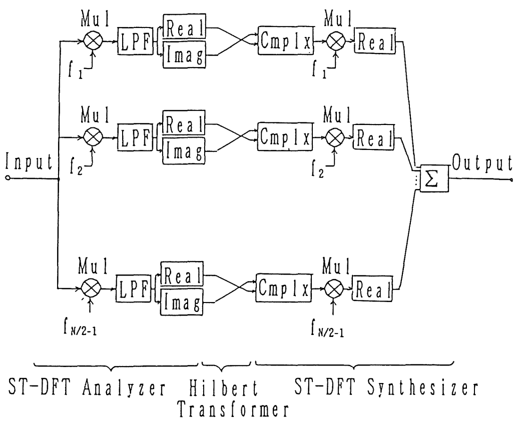

Fig. 1 Circuitry configuration of the short time DFT Hilbert transformer

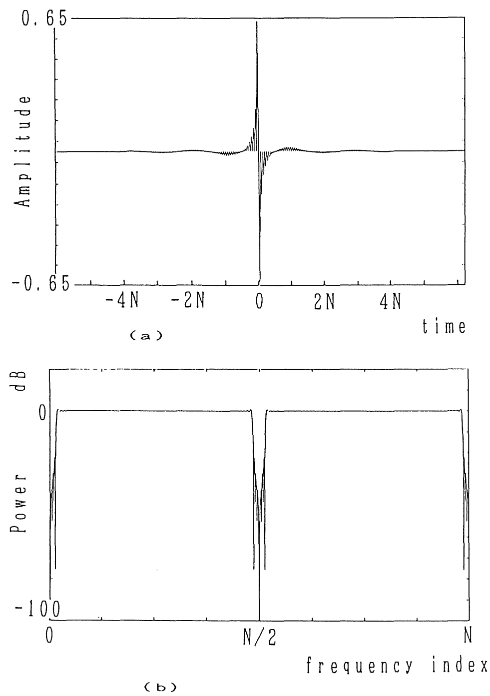

Fig. 2 Unit sample response of the short time DFT Hilbert transformer: (a) time domain response, and (b) its power spectrum

Hilbert transformer is based on that the Hilbert transformation is precisely performed on the frequency domain merely by exchanging real parts with imaginary parts of instantanuous spectrums in the input signal and that the exact Hilbert transformed signals are yielded through synthesizing appropriate signals from instantanous spectrums consist of the exchanged real and imaginary parts.

The significant instantaneous spectrum analysis and synthesis in the exact Hilbert transformers are covered by the short time DFT and short time IFT. This new type Hilbert transformer proposed in this paper, named by "short time DFT Hilbert transformer", is strictly substantiated by both phase shift characteristics being $\pi/2$ radians and amplitude being unity through computer simulations.

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