Effect of Creep on Stability of Reinforced Concrete Column

Zhang HENGPING, Tetsuhiko AOKI and Shunji INOMATA

クリープによる鉄筋コンクリート柱の安定性解析

張　恒平・青木徹彦・猪股俊司

In this paper the second-order effect is evaluated by taking into account th influence of creep correnponding to the working loads. The moment-curvature relations at time t are calculated from the equilibrium equations for axial load and moment, and the assumption of linear strain distribution (plane-section assumption). In this case, the stress-strain diagram for concrete is modified by multiplying the short-term strains by a factor $(1+\phi)$, in order to take account the effect of creep. A program for determining N−M−1/R relationships is developed, and a rational approach is proposed to investigate the creep stability failure of reinforced concrete columns.

1. INTRODUCTION

A column subjected to eccentrically applied axial load will deflect laterally. This deflection may significantly affect the distribution and magnitude of internal forces and consequently also the load bearing capacity of the column. If the load is sustained, the deflection will increase further due to creep of comcrete.

Under sustained external forces, the initial deflection, and consequently the moment, is magnified due to creep. If the external forces are relatively small in comparison with the bearing capacity of the column, the creep deflection will terminate at some finite value and an equilibrium state is reached between external and internal forces at all sections constituting the column. Furthermore, at each section, the curvature due to internal stress is then equal to the curvatur of the deflection line.

At high external forces, the creep deflection may increase continuously until the column fails. Depending upon the column geometry (mainly slenderness and eccentricity), the failue load may be reached either at material failure (typical for shorter columns) or at stability failue (typical for more slender columns). Material failue is associated with crucking of the concrete or excessive yielding of the reinforecement. When failure is due to instability, the material is not loaded to the limit of its strength.

This paper deals with the effect of creep on the behaviour of column subjected to sustained loading and proposes a rational approach to investigate the creep stability failure of the column.

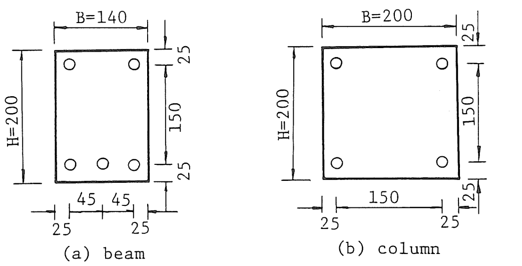


Fig. 1 Cross Sectional Dimensions of RC Members

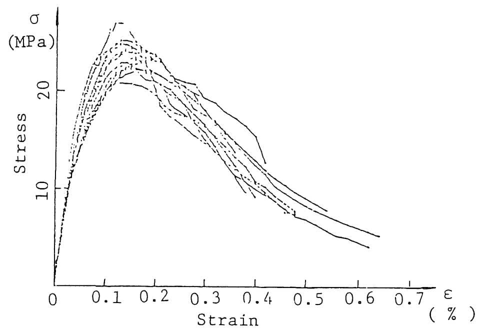


Fig. 2 Stress-Strain Diagram for Concrete Subjected to Axial Compression

2. N-M-1/R RELATIONS WITHOUT EFFECT OF CREEP

The numerical program for calculating N-M-1/R relations is almost the same as that for the inelastic bending analysis of RC beams. The program developed in this study was ascertained by comparing the results of RC beam bending tests. The cross sectional dimensions of specimens in the test are shown in Fig.

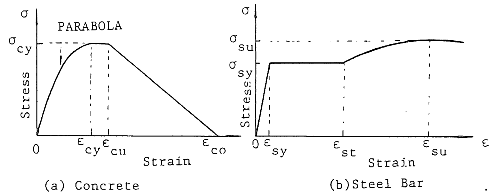


Fig. 3 Idealized Stress-Strain Curves for Concrete and Reinforcing Bar

Table 1 Results of Concrete Compression Test(N=14)

	σcy (MPa)	(x 10^{-3}) εcy	εcu	εco
M	22.5	2.32	3.06	13.8
S	1.47	0.15	0.08	0.4

Table 2 Results of Steel Bar Tension Test(N=12)

	σsy (MPa)	σsu (MPa)	(x 10^{-3}) εsy	εst	εsu
M	388	524	1.89	18.5	66.3
S	9.6	9.7	0.05	0.0	0.0

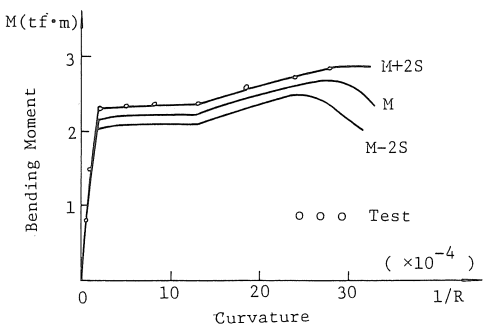
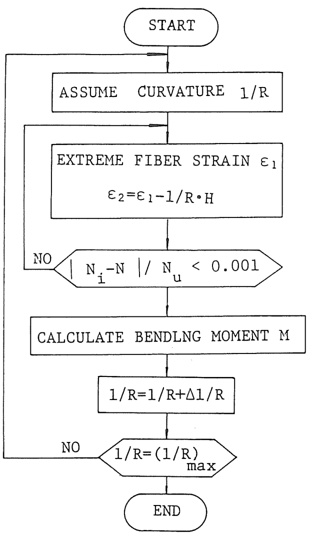


Fig. 5 M -1/R Curve by Test and Calculation

1 (a) and the same dimensions were used for the numerical analysis. Fourteen cylindrical specimens (D=100mm, h=200mm) of the same concrete material as used for beams were tested, yielding the stress-strain diagrams shown in Fig. 2. The deformed steel bars used in the beams were all of the same steel grade with SD 35 (nominal yield strength 350Mpa) and size D 13 (nominal cross sectional area 1.267cm^2). Twelve specimens were used for the tensile test. The idealised stress-strain curves indicated in Fig. 3 were applied to each test result in order to determine the material constants in the figures. These data were summarized in Table 1 and 2, in which M is the mean average and S the standard deviation.

In the calculation of N-M-1/R relations, the tensile stress in the concrete members was neglected and the law of reservation of plane was assumed. By first assuming curvature 1/R and the assigning a certain value for one extreme fiber strain $\varepsilon 1$, the opposite extreme fiber strain $\varepsilon 2$ can be found by trial, given that the axial stress resultant N has to be zero. The flow chart for the calculations of N-M-1/R is shown in Fig. 4.

Numerical calculations for beams were performed using material data M and M±2S from Tables and compared with the results of the beam bending test. The solid curves in Fig. 5 indicate numerical results and the round marks indicates the test results, which coincided with the M+2S curve, being 6% above the M curve. It may be said that a close agreement was observed for the whole M-1/R curve between the test results and those obtaied from calculation. One conceivable reason for the slightly greater moment found in the test result might be the increasing strength of the steel bars embedded in the concrete of the beam.



WHERE

H: HEIGHT OF CROSS SECTION
N: AXIAL APPLIED FORCE
N_i: AXIAL INTERNAL FORCE
N_u: MAXIMUM AXIAL FORCE
$=Ac \cdot \sigma_{cy} + As \cdot \sigma_{sy}$

Ac, As : CROSS SECTIONAL AREA CONCRETE AND STEEL BAR, RESPECTIVELY

Fig. 4 Calculatlon Flow Chart of N-M-1/R Relation

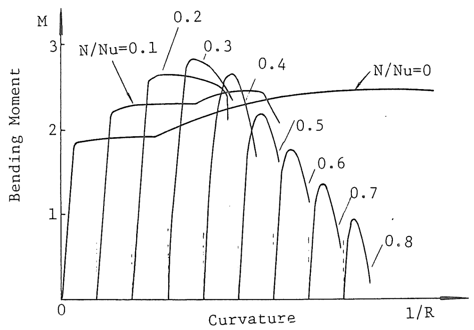


Fig. 6 N – M – 1/R Curves for Various Axial Force N/Nu

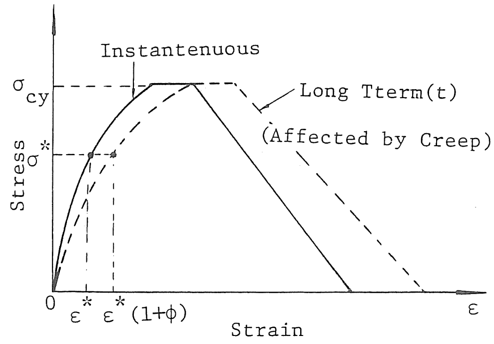


Fig. 7 Stress – Strain Diagram of Concrete Affected by Creep

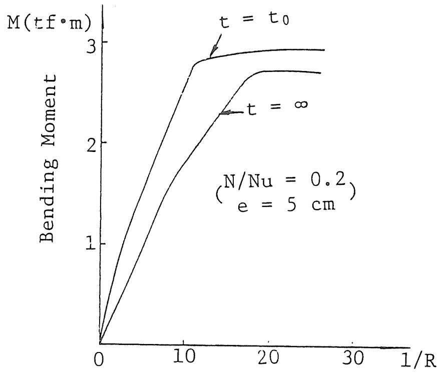


Fig. 8 Example of N-M-1/R Curves Taking into account of Creep(t=∞)

Example of numerical calculation of N-M-1/R relations for column sections (Fig. 1 (b) $A_c=400cm^2$, $A_s=12.16cm^2$) are presented in Fig. 6. The material constants used in calculation are the same as those used for the beam. The basic axial force N_u in Fig. 6 is so chosen that the uniform strain ε_{cy} occurring in the member exactly corresponds to maximum concrete strength σ_{cy}. At this moment the stress in the steel bars becomes the yield strength σ_{sy}. N_u can then be expressed as in Eq. (1)

$$N_u=A_c\cdot\sigma_{cy}+A_s\cdot\sigma_{sy} \qquad [1]$$

where the notation A_c and As are used as indicated in Fig. 4. Non-dimensionalized external axial force N/N_u changes at step 0.1 from 0.1 to 0.8 in this example. Although the maximum moment increases with N/N_u up to 0.3, rotational capacity decreases markedly in spite of the small addition of axial force. For axial force N/N_u beyond 0.3, both maximum moment and rotational capacity decrease.

3. EFFECT OF CREEP ON N-M-1/R RELATIONS

In order to take account the effect of creep on N-M-1/R relations, the stress-strain diagram for concrete is modified by multiplying the short-term strains by a factor $(1+\phi)$, as shown in Fig. 7, where ϕ is creep coefficient.

The creep coefficient ϕ (t, t_0) can be determined from :

$$\phi(t, t_0)=\beta a(t_0)+\phi d\cdot\beta d(t-t_0) + \phi f[\beta f(t)-\beta f(t_0)]$$

where :

$\beta a(t_0)/Ec_{28}$ = rapid initial deformation which is developed during the first day after the load has been imposed,

$\phi d\cdot\beta d(t-t_0)/Ec_{28}$ = recoverable part of the delayed deformation (delayed elasticity) assumed to be independent of aging in its development and characterized by a constant value of the coefficient ϕd,

$\phi f[\beta f(t)-\beta f(t_0)]/Ec_{28}$ = irreversable delayed deformation (flow) which is very much affected by the age at which loading commences,

t, t_0 = age of concrete at the moment under consideration and of loading, respectively,

βd, βf = function corresponding to the development with time of the delayed elastic strain and the flow strain, respectively.

The creep coefficient depends upon the ambient. humidity, the dimensions of the element, and the time t and t_0. In order to allow the recording of the model in the computer calculation, analytical expressions shown in the "CEB Manual on Structural Effect of Time -dependent Behaviour of Concrete (1984)"[2] are adopted in this paper.

A computed M-1/R diagram for a given normal force is shown in Fig. 8.

4. ITERATION PROCEDURE FOR DETERMINATION OF STABILITY

The column is divided in a finite number of elements, and the deflection of each element is approximated by a circle. The curvature of segment is determined by interpolation, finding the point on the previously calculated N-M-1/R curve that corresponds to the external moment at the center of the segment. Fig. 9 shows one part of the column deformed to an approximate arc with local coordinate x-y. The global coordinates for the column are expressed as X and Y.

Using coordinates (Xi, Yi) to define point A on the one end of the arc, and coordinate (Xi+ΔXi, Yi+ΔYi) to define point B separated from A by the arc length Δl, we find the following relation;

$$\begin{Bmatrix}\Delta X\\ \Delta Y\end{Bmatrix} = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{Bmatrix}b\\ \Delta \ell\end{Bmatrix} \simeq \begin{bmatrix}1 & \alpha\\ -\alpha & 1\end{bmatrix}\begin{Bmatrix}b\\ \Delta \ell\end{Bmatrix} \quad [4]$$

where b is the x-coordinate of point B on the x y axis. Assigning $\angle ABC=\theta$ and the radius of curvature R, θ and b can be expressed as follows:

$$\theta=\Delta l\cdot(1/R), \quad b=\Delta l\cdot\theta/2=(\Delta l)^2\,(1/2R) \quad [5]$$

Therefore, if the curvature 1/R of this arc is obtainable from the N-M-1/R relation, b may be determined from Eq. (5) and ΔX, ΔY from Eq. (4).

Assembling the segment into the column again in such a way that the inclination on the tangents of adjacent segments becomes equal, the shape of the column can be determined. The external moment at each segment may be calculated from the column's deflection and degree of eccentricity, and the corresponding curvature 1/R may be found from the N-M-1/R curves. Since each segment has a slightly different curvature from the former stage, the column's shape after reassembling will also become slightly different. Repeating this procedure, the column's shape will tend to an converge to an equilibrium position.

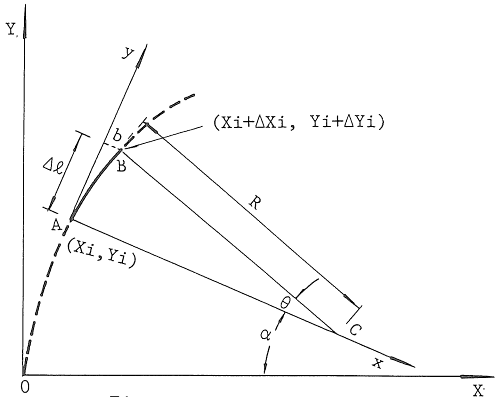


Fig. 9 Calculation of Column Deflecion

5. NUMERICAL EXAMPLES

Using above mensioned procedure, the column deflection δ versus time t relation is calculated. Fig. 10 shows one example of numerical analysis for a RC column with the cross section illustrated in Fig. 1 (b), lenght 3 m and eccentricity e=5cm. When axial force N/Nu was changed from 0.2 to 0.3 by step 0.02, it was found that at N/Nu= 0.26 the column remained stable, but that at N/Nu= 0.28 it became unstable. Accordingly, N/Nu was divided into four values in this region and the δ — t relation was once again investigated, giving the broken lines in Fig. 10.

From these results, the critical axial force N/Nu of this column may be assumed to lie between 0.270 and 0.275. In this way, if the relation between time and column deflection dependant creep is calculated and the maximum sustainable axial load of a column with eccentricity can be ascertained.

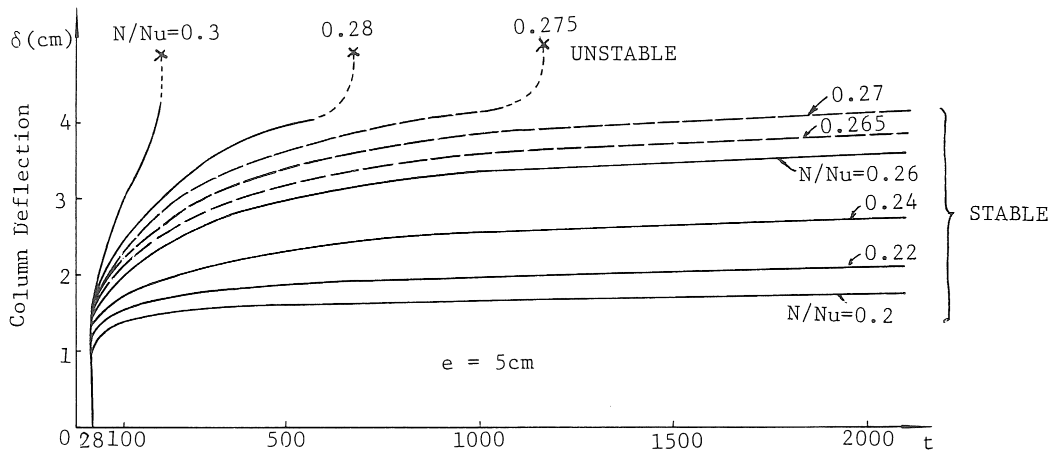


Fig. 10 Column Deflection δ and Time t Relationship

6. CONCLUDING REMARKS

The effect of creep on the stability of reinforced concrete columns has been analyzed by a rational approach. The method assumes that concrete creep strain is proportional to stress under warking load conditions and the stress-strain diagram at time t can be obtained by modifying the short-term diagram which strain values are multiplied by $(1+\phi)$. It can be used with any chosen functions expressing the time variation of concrete creep.

The column examples analyzed show that accurate evaluation of stability of reinforced concrete columns cannot be done without accounting for the effect of concrete creep.

References

(1) Kato. M, Aoki. T, Fuwa. A," Experimental study of ultimate bending bearing capacity of RC continuous beams", Annual Sympo. of JSCE Chubu Branch, March, 1985

(2) CEB Manual on Structural Effect of Time-dependent Behaviour of Concrete, 1984

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