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On the Riesz-type Decomposition Theorem and its Applications in Potential Theory of Function-Kernels

関数核ポテンシャル論における リース型分解定理とその応用について

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Abstract

In this paper we shall first obtain a Riesz-type decomposition theorem of super-harmonic functions with respect to function-kernels of potentials.

Next, as an application of new decomposition theorem, we shall give some new characterizations of the regularity of function-kernels which plays an important role in the theory of Hunt kernels.

1. Introduction

Let X be a locally compact but non-compact Hausdorff space satisfying the second axiom of countability.

A positive linear mapping from C_K to C is called a **continuous kernel on** X .

The family $(V_p)_{p>0}$ of continuous kernels on X is colled a **resolvent family associated with** V , if it satisfies the following equalities:

$$(3) \quad V_p - V_q = (q-p)V_p \cdot V_q = (q-p)V_q \cdot V_p, \qquad \forall p,\ \forall q \ > \ 0 \ ,$$

$$(4) \quad \lim_{p\to 0} V_p = V \ .$$

G.A.Hunt[11] verified that, when a continuous kernel V satisfies the complete maximum principle, we can associate a resolvent family $(V_p)_{p>0}$ with V , under the assumption that $V(C_K) \subset C_0$ and $V(C_K)$ is dense in C_0 .

The existence of a resolvent family may be developped to the theory of a semi-group. So we can consider a continuous kernel as the elementary solution of the infinitesimal generator of the semi-group and hence we can enter analytically into the arguement of the generalized Poisson and Dirichlet problems.

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Subsequently, G.Lion[20] obtained the same result without the condition that $V(C_K)$ is dense in C_0 .

On the other hand, P.A.Meyer[21], J.C.Taylor[22] and F.Hirsch[10] constructed the resolvent family replacing the condition that $V(C_K) \subset C_0$ with the weaker conditions on the vanishing properties of potentials at infinity.

The weaker condition above mentioned is called the **regularity** of kernels in potential theory.

Now, let us recall here the arranged results in the theory of convolution kernels on a locally compact abelian group X

A convolution kernel N on X is called a **Hunt kernel** when there exists a vaguely continuous semi-group $(\alpha_t)_{t>0}$ of positive measures on X satisfying

$$N = \int_0^\infty \alpha_t \, dt \quad \Big(\text{i.e., } \int f \, dN = \int_0^\infty \Big\{ \int f \, d\alpha_t \Big\} \, dt \ \text{ for } \forall f \in C_K \Big).$$

Concerning the characterization of Hunt kernel, the following rerults are well known.

A non-periodic convolution kernel N becomes a Hunt kernel if and only if N satisfies one of the following conditios:

(A) N is **balayable** ,that is, there exists a balayaged measure on every open set not necessarily relatively compact (cf. G.Choquet-J.Deny[1]).

(B) There exists a **resolvent family associated with** N (cf. M.Itô[12]).

(C) N satisfies the domination principle and N is **regular** (cf. M.Itô[12]).

(D) N satisfies the domination principle and has **the dominated convergence property** (cf, M.Itô[12] and M.Kishi[19]).

Remark 1. In the theory of continuous function-kernels, the author has investigated the relations $(A) \sim (D)$ and he has already verified the equivalence of (C) and (D) and obtained the relations $(C) \longrightarrow (A)$ and $(C) \longrightarrow (B)$ (cf. I.Higuchi[5], [6], [7]).

But the inverse relations $(A) \longrightarrow (C)$ and $(B) \longrightarrow (C)$ fail to hold in general (cf. I.Higuchi[7] and M.Itô[14]).

These facts suggest that the treatments of the function-kernels are more complicated than that of the convolution kernels.

The regularity of function-kernel is concerned deeply with the vanishing property of potentials in the neighbourhood of the point at infinity.

Indeed, the author proved that a continuous function-kernel $G = G(x, y)$ is **regular** if and only if at least one of G and its adjoint $\check{G}$ converges to 0 quasi-everywhere at infinity in the case that both G and $\check{G}$ satitfy the complete maximum principle (cf. I.Higuchi[9]).

The purpose of this paper is to obtain a **Riesz-type new decomposition theorem** of superharmonic function with respect to a continuous function-kernel G and to prove that G is **regular** if and only if both G and $\check{G}$ are **reduction regular** , in the case that G satisfies the domination principle.

2. preliminaries

Let X be a locally compact but non-compact Hausdorff space satisfying the second axiom of countability. A function $G = G(x,y)$ on $X \times X$ is called **a continuous function-kernel** on X when it satisfies

$$0 < G(x,x) \leq +\infty \quad \text{for } \forall x \in X \ ,$$

$$0 \leq G(x,y) < +\infty \quad \text{for } \forall (x,y) \in X \times X \text{ with } \ x \neq y \ .$$

The G-potential $G\mu(x)$ of a Radon mesure μ on X is defined by

$$G\mu(x) = \int G(x,y) \ d\mu(x).$$

Put

$$M = \{ \ \mu : \text{positive Radon mesure on } X \ \},$$

$$E = E(G) = \{ \ \mu \in M \ ; \ \textstyle\int G\mu(x) \ d\mu(x) < +\infty \ \},$$

$$F = F(G) = \{ \ \mu \in M \ : G\mu(x) \text{ is finite continuous on } X \ \},$$

$$D = D(G) = \{ \ \mu \in M \ : \ G\mu(x) < +\infty \ \text{ G-n.e. on } X \ \}.$$

And we write their sub-families consisting of the measures with compact support by M_0, E_0, F_0 respectively.

We denote by $P_{M_0}(G)$ the totality of G-potentials of the measures in M_0. The notations of the families of various class of potentiala are also denoted similarly.

A Borel measurable set B is said to be G**-negligible** if $\mu(B) = 0$ for $\forall \mu \in E_0(G)$. We say that a property P holds G-nearly everywhere on a subset A of X and write simply that P holds G**-n.e.** on A, when it holds on A except for a G-negligible set.

A lower semi-continuous function u on X is said to be **G-superharmonic** when $0 \leq u(x) < +\infty$ G-n.e. on X and for any $\mu \in E_0(G)$, the inequality $G\mu(x) \leq u(x)$ G-n.e. on $S\mu$ implies the same inequality on the whole space X .

We denote by $S(G)$ the totality of G-superharmonic functions on X.

For a function $u \in S(G)$ and a closed set $F \subset X$, a positive measure μ' supported by F satisfying the following conditions is called a balayaged mesure of u on F, if it exists:

$$G\mu'(x) = u(x) \quad \text{G-n.e. on} \quad F \ ,$$

$$G\mu'(x) \leq u(x) \quad \text{on} \quad X \ .$$

We denote by $S_{bal}(F,G)$ the totality of G-superharmonic functions for which the balayaged nesure on F exists and write simply $S_{bal}(G)$ instead of $S_{bal}(X,G)$.

Potential theoretic principles are stated as follows:

(i) We say that G satisfies **the domination principle** and write simply $G \prec G$ when $P_{M_0}(G) \subset S(G)$.

(ii) We say that G satisfies **the complete maximum principle** and write simply $G \prec G+1$ when we have

$$P_{M_0}(G) \cup \{c\} \subset S(G) \quad \text{for } \forall c \geq 0.$$

(iii) We say that G satisfies **the balayage principle** when we have

$$P_{M_0}(G) \subset \cap_{k;compact \subset X} S_{bal}(K, G).$$

(iv) We say that G is **balayable** when we have $P_{M_0}(G) \subset S_{bal}(G)$.

(v) We say that G satisfies **the continuity principle** if, for any $\mu \in M_0$, the finite continuity of the restriction og $G\mu(x)$ to $S\mu$ implies the finite continuity of $G\mu(x)$ on the whole space X.

When a continuous function-kernel G satisfies **the continuity principle**, we can verify, under the additional condition that every non-empty open set in X is of positive G-inner capacity, that there exists a positive mesure ξ everywhere dense on X satisfying

(1) $G(x,y)$ is locally $\xi \otimes \xi$ -summable,

(2) $V_G^{\xi}(f)(x) = \int G(x,y) f(y)\, d\xi(y)$ is continuous on X for $\forall f \in C_K$.

Then we can consider V_G^{ξ} as **a continuous kernel** on X .

For a non-negative Borel function u and a closed set F, **the G-reduced function of** u **on** F and **the G-reduced function of** u **on** F **at infinity** δ , are defined respectively by

$$R_G^F(u)(x) = \inf \{ \ v(x) \ ; \ v \in S(G), \ v(x) \geq u(x) \ G\text{-}n.e. \text{ on } F \ \},$$

$$R_G^{F,\delta}(u)(x) = \inf_{\omega \in \Omega_0} R_G^{F \cap C\omega}(u)(x),$$

where Ω_0 denotes the totality of all relatively compact open sets in X.

And we write simply $R_G^{\delta}(u)(x)$ instead of $R_G^{X,\delta}(u)(x)$.

Put, for a closed set F,

$$S_0(F,G) = \{ \ u \in S(G) \ ; \ R_G^{F,\delta}(u)(x) = 0 \ G\text{-}n.e. \text{ on } X \ \},$$

And write simply $S_0(G)$ instead of $S_0(X,G)$.

Remark 2. When G satisfies the domination principle, the following (1) and (2) hold:

(1) We have $R_G^F(u) \in S(G)$ for any closed set F and for any $u \in S(G)$.

(2) We have $\hat{R}_G^\delta(u) \in S(G)$ for any $u \in S(G)$, where $\hat{R}_G^\delta(u)(x)$ denotes the lower regularization of $R_G^\delta(u)(x)$.

Further we put, for a closed set F ,

$$S_0(F, G) = \{ \ u \in S(G) \ ; \ R_G^{F,\delta}(u)(x) = 0 \ G\text{-}n.e. \text{ on } X \ \},$$

and write simply $S_0(G)$ instead of $S_0(X, G)$.

The kernel G is said to be **regular** when we have $P_{M_0}(G) \subset S_0(G)$.

Remark 3(cf. I.Higuchi[7] and M.Itô[15]). When G satisfies the domination principle, the following statements are equivalent:

(1) G is regular.

(2) $P_{E_0}(G) \ \subset \ S_0(G)$.

(3) $P_{F_0}(G) \ \subset \ S_0(G)$.

(4) $P_{D_0}(G) \ \subset \ S_0(G)$.

(5) $\check{G}$ is regular.

Therefore, it suffices to obtain the weakest condition (3) when we show the regulariry of G and we may use the strongest condition (4) when we apply the regularity of G.
And the duality of regularity follows from the equivalence of (1) and (5).

Remark 4. Suppose that G satisfies the complete maximum principle and that, for $\forall \mu \in M_0$, $G\mu(x)$ converges uniformly to 0 at infinity δ , that is, for $\forall \epsilon > 0$ and for $\forall \mu \in M_0$,there exists an $\omega \in \Omega_0$ satisfying $G\mu(x) < \epsilon$ on $C\omega$. Then G becomes regular. Therefore, regularity means a kind of vanishing property of potentials at infinity δ .

Remark 5(cf. I.Higuchi[8],[9]). We have already generalized Remark 4 and characterized the regularity as follows:
Suppose that both G and $\check{G}$ satisfy the complete maximum principle. Then the following (1) and (2) are equivalent each other:

(1) G is **regular** .

(3) For $\forall c > 0$, $\forall d > 0$, $\forall \mu$, and for $\forall \nu \in M_0$, we have

$$\inf_{\omega \in \Omega_0} cap_G^i \Big[\ \{ \ x \in X \ ; \ G\mu(x) \geq c \ \} \ \cap \ \{ \ x \in X \ ; \ \check{G}\nu(x) \geq d \ \} \Big] \ = \ 0 \ .$$

3. Riesz-type decomposition theorem

In the rest of this paper, we discuss always on the following assumption:

every non-empty open set in X is of positive G-inner capacity .

After the classical model of Riesz, we shall have a following new decomposition theorem of superharmonic functions with repect to continuous function-kernels.

Theorem 1. *Let G be a continuous function-kernel on X satisfying the domination principle.*

Then for any $u \in S(G)$ and for any closed set $F \subset X$, there exist a positive measure $\mu_F \in D(G)$ and a function $h_F(x)$ such that

$$\begin{aligned}
u(x) &= G\mu_F(x) + h_F(x) \quad on \quad X, \\
& S\mu_F \subset F, \\
& \exists \{\mu_n\}_{n=1}^{\infty} \subset E_0(G) \quad s.t. \\
& \qquad S\mu_n \subset F \quad for \ \forall n \ , \\
& \qquad \mu_n \longrightarrow \mu \ (vaguely) \ as \ n \longrightarrow \infty \ , \\
& \qquad G\mu_n(x) \leq G_{n+1}(x) \quad on \quad X \quad for \quad \forall n \ , \\
& \qquad \lim_{n\to\infty} G\mu_n(x) = u(x) \quad G\text{-}n.e. \quad on \quad F. \\
& G\mu_F(x) \leq u(x) \quad on \quad X, \\
& h_F(x) \leq R_G^{F,\delta}(u)(x) \quad G\text{-}n.e. \quad on \quad F.
\end{aligned}$$

Definition The sequence $\{\mu_n\}_{n=1}^{\infty} \subset E_0(G)$ (resp. the measure $\mu_F \in D(G)$) is called an **approximate sequence of balayaged mesure** (resp. a **pseudo-balayaged measure**) of u on F .

Remark 6. The proof of the classical decomposition theorem of Riesz concerning the superharmonic functions on R^n $(n \geq 3)$ is done by using the relations held between the Laplace operator Δ and the Newton kernel $N = N(x, y)$.

And by virtue of the celebrated lemma of Weyle, we can prove the harmonicity of the function h_F .

On the other hand, in our new decomposition theorem, the generalized Laplacian with respect to the kernel G does not appear on the stage.

So we can not derive the harmonicity of function h_F in theorem 1 .

But, approximating a G-superharmonic function u by the potential $G\mu_F(x)$, we may appreciate the function $h_F(x)$ by the G-reduced function $R_G^{F,\delta}(u)(x)$ on F .

Therefore, we can investigate the behavior of u in the neighbourhood of the point at infinity.

For the proof of Theorem 1 , we recall here the following two lemmas and the equivalence held between the relative domination principle and the relative balayage principle(cf. M.Kishi[18] and I.Higuchi[4]).

Lemma 1(cf. R.Durier [3]). *Let G be a continuous function-kernel on X such that its adjoint $\check{G}$ satisfies the continuity principle and $\{\mu_n\}_{n=1}^{+\infty}$ be a sequence of measures in $D(G)$.*
Suppose that there exists a superharmonic function $u \in S(G)$ satisfying

$$G\mu_n(x) \ \leq \ u(x) \ \ G\text{-n.e. on } X \ \ \text{for } \forall n \ .$$

Then, the set $\{\mu_n\}_{n=1}^{+\infty}$ is vaguely bounded.

Lemma 2(cf. I.Higuchi[7]). *Suppose that a continuous function-kernel G satisfies the domination principle. Then there exists, for any $u \in S(G)$ and for any clode set F in X , a sequence $\{\mu_n\}_{n=1}^{+\infty} \subset E_0(G)$ of measures verifying*

$$S\mu_n(x) \ \subset \ F \ \ \text{for } \forall \ n,$$

$$G\mu_n(x) \ \leq \ G\mu_{n+1}(x) \ \leq \ u(x) \ \ \text{on} \ \ X,$$

$$\lim_{n\to\infty} G\mu_n(x) \ = \ u(x) \ \ G\text{-n.e.} \ \ \text{on} \ \ F \ .$$

Proof of theorem 1. We denote by $\{\Omega_n\}_{n=1}^{+\infty}$ the exhaustion of X such that Ω_n is a relatively compact open set in X . Put

$$F_n = F \cap \overline{\Omega}_n \cap \{x \in X; u(x) \leq n\} \ .$$

Then F_n converges increasingly to F as n tends to $+\infty$.
Let $\mu_F \in D(G)$ be a pseudo-balayaged measure of u on F . We may concider $\{\mu_n\}_{n=1}^{+\infty} \subset E_0(G)$ as an approximate sequence of balayaged mesure of u on F and μ_F as a vague adherent of $\{\mu_n\}_{n=1}^{+\infty}$.
We denote by $\mu_{m,n}$ the restriction of μ_n to $C\Omega_m$.
Then we have

$$G\mu_{m,n}(x) \leq G\mu_n(x) = u(x) = R_G^{F\cap C\Omega_m}(u)(x) \quad \text{on } S\mu_{m,n} \text{ and hence on } X$$

and, by the inequality $\mu_n \geq \mu_n - \mu_{m,n}$,

$$\begin{aligned} G\mu_F(x) &\geq \liminf_{n\to\infty}\{G\mu_n(x) - G\mu_{n,m}(x)\} \\ &\geq \lim_{n\to\infty} G\mu_n(x) - R_G^{F\cap C\Omega_m}(u)(x) \\ &= u(x) - R_G^{F\cap C\Omega_m}(u)(x) \quad G\text{-n.e. on } F \ , \end{aligned}$$

and therefore, letting n tend to $+\infty$,

$$h_F(x) = u(x) - G\mu_F(x) \leq R_G^{F,\delta}(u)(x) \quad G\text{-n.e. on F} \ .$$

This completes the proof of Theorem 1 .

4. Applications of Riesz-type decomposition theorem

The following result concerning the relation held between the regularity and the balayability is an immediate consequence of Theorem 1 .

Corollary 1. *If G satisfies the domination principle, the following inclusion relation holds:*

$$S_0(G) \subset S_{bal}(G).$$

Remark 7. The relation in Corollary 1 was already obtained by using the equivalence held between the regularity and the so-called **dominated convergence property** (cf. I.Higuchi[5]). We emphasize here that Corollary 1 follows immediately from Theorem 1 .

Remark 8 . The inverse inclusion relation of Corollary 1 does not necessarily hold in general. But if we suppose that G satisfies the domination principle and that G is regular, then we have $S_0(G)=S_{bal}(G)$ and thereforethe, following (1) $\sim$ (4) are equivalent:

(1) $u \in S_0(G)$.

(2) $u \in S_{bal}(G)$.

(3) $\hat{R}^{\delta}_G(u) \in S_0(G)$.

(4) $\hat{R}^{\delta}_G(u) \in S_{bal}(G)$.

Remark 9 . By Corollary 1 , we may discuss the vanishing property of superharmonic function G at infinity point using the balayability of u . Further, we may derive the the vanishing property of u at infinity point from that of the smaller function $R^{\delta}_G(u)(x)$.

Corollary 2(cf. I.Higuchi[7]). *When $G \prec G$, G is balayable if G is regular.*

Remark 10 . The inverse of Corollary 2 does not correct in general.

In fact, We denote by $N = N(x,y)$ the Newton kernel on $R^n (n \geq 3)$ and by ξ a positive measure such that $N\xi(x)$ is finite continuous on X and that $\int d\xi < +\infty$.

The the continuous function-kernel defined by

$$G(x,y) = N(x,y) + N\xi(x)$$

satisfies the domination principle. Further we can prove that G is balayable but not regular and hence that the regularity is a stronger property than the balayability(cf. I.Higuchi[7]).

Now we consider the three definitions of regularity and obtain the mutual relations held in them.

Theorem 2. *For the contonuous function-kernel G satisfying the domination principle, the following* (1) $\sim$ (4) *are equivalent each other:*

(1) *G is* **regular**, *that is, the following inclusion holds:*

$$P_{M_0}(G) \ \subset \ S_0(G) \ .$$

(2) *$\check{G}$ is* **regular**, *that is, the following inclusion holds:*

$$P_{M_0}(\check{G}) \ \subset \ S_0(\check{G}) \ .$$

(3) *Both G and $\check{G}$ are* **reduction regular**, *that is, the following inclusions hold at the same time:*

$$\hat{R}^{\delta}_{G}(P_{M_0}(G)) \subset S_0(G) \ ,$$

$$\hat{R}^{\delta}_{\check{G}}(P_{M_0}(\check{G})) \subset S_0(\check{G}) \ .$$

(4) *Both G and $\check{G}$ are* **strongly regular** , *that is, every G-***pseudo-potential** *(resp. every $\check{G}$-***pseudo-potential***) is contained in $S_0(G)$ (resp. in $S_0(\check{G})$) .*

Remark 11 . A G-superharmonic function u is called a G-pseudo-potential when u is dominated by a potential $G\mu(x)$ of some measure $\mu \in M_0(G)$.

Corollary . *Let G be a symmetric continuous function-kernel on X . Then the following three statements are equivalent each other:*

(1) *G is* **regular** .

(2) *G is* **reduction regular** .

(3) *G is* **strongly regular** .

Proof of Theorem 2 . The equivaleces (1) $\longleftrightarrow$ (2) $\longleftrightarrow$ (4) have been already known (cf. Remark 3 and I.Higuchi[8]).

By virtue of the inequality $u(x) \geq R^{\delta}_{G}(u)(x)$ on X the implication (1) $\longrightarrow$ (3) is trivial.

Therefore, we may suffice to verify the inverse implication (3) $\longrightarrow$ (1) .

To prove the implication (3) $\longrightarrow$ (1) , we use the Riesz-type decomposition theorem obtained in Theorwm 1 .

For any $\mu \in M_0(G)$ and any $\omega_0 \in \Omega_0$, we denote by $\mu_{C\omega_0}$ a pseudo-balayaged measure of μ on $C\omega_0$.

Then we have first, for any measure $\nu \in F_0(\check{G})$, the following equality when both G and $\check{G}$ are reduction regular:

$$\int R_G^\delta(G\mu)(x)d\nu(x) = \int R_G^\delta(G\mu_{C\omega_0})(x)d\nu(x) .$$

In fact, we have by (3),

$$\int R_G^\delta(G\mu)d\nu \le \lim_{\omega\to X} \int R_G^{C\omega}\{G\mu_{C\omega} + R_G^\delta G(\mu)\}d\nu$$

$$= \lim_{\omega\to X} \int R_G^{C\omega}(G\mu)d\nu + \int R_G^\delta(R_G^\delta(G\mu))d\nu$$

$$= \lim_{\omega\to X} \int G\mu_{C\omega}d\nu + 0 = \lim_{\omega\to X} \int \check{G}\nu d\mu_{C\omega}$$

$$= \lim_{\omega,K\to X} \int R_{\check{G}}^{C\omega\cap K}(\check{G}\nu)d\mu_{C\omega} = \lim_{\omega,K\to X} \int \check{G}\nu_{C\omega\cap K}d\mu_{C\omega}$$

$$= \lim_{\omega,K\to X} \int G\mu_{C\omega}d\nu_{C\omega\cap K} \le \lim_{\omega\to X} \int R_G^{C\omega}(G\mu_{C\omega_0})d\nu$$

$$= \int R_G^\delta(G\mu_{C\omega_0})d\nu .$$

Consequently we have the following inequality:

$$\int R_G^\delta(G\mu)d\nu \le \int R_G^\delta(G\mu_{C\omega_0})d\nu .$$

The inverse inequality being trivial, we obtain the desired equality:

$$\int R_G^\delta(G\mu)(x)d\nu(x) = \int R_G^\delta(G\mu_{C\omega_0})(x)d\nu(x) .$$

In the above calculations, we used the theorem of Fubini and the dominated convergence theorem of Lebesgue repeatedly.

Finally we have

$$\int R^{\delta}_{G}(G\mu)d\nu = \int R^{\delta}_{G}(G\mu_{C\omega_0})d\nu = \int R^{\delta}_{\check{G}}(\check{G}\nu)d\mu_{C\omega_0}$$

$$\leq \int R^{C\omega_0}_{\check{G}}(\hat{R}^{\delta}_{\check{G}}(\check{G}\nu))d\mu_{C\omega_1} \quad (\omega_1 \in \Omega_0 \ \ and \ \ \omega_1 \subset \omega_0) .$$

Letting ω_0 tend to X , we have

$$\int R^{\delta}_{G}(G\mu)d\nu \leq \int R^{\delta}_{\check{G}}(\hat{R}^{\delta}_{\check{G}}(\check{G}\nu))d\mu_{C\omega_1} = 0 .$$

The last equality follows from our assumption that $\check{G}$ is also reduction regular.
Consequently, the implication (3) $\longrightarrow$ (1) was verified and hence the proof of Theorem 2 was completed .

References

[1] G.Choquet-J.Deny, Noyaux de convolution et balayage sur tout ouvert, Lecture Notes in Math.,Springer-Verlag, No. 404(1973), 60-112.

[2] J.Deny, Noyaux de convolution de Hunt et noyaux associées à une famille fondamontale, Ann.Inst Fourier, 12(1962), 643-667.

[3] R.Durier, Sur les noyaux-fonctions en théorie du potentiel, Rend. Corc. Mat. Palermo, (2), 13(1969), 113-180.-

[4] I.Higuchi, Duality of domination principle for non-symmetric lower semi-continuous function-kernels, Hiroshima Math.J., 5(1975), 551-559.

[5] ——, Régularité et propriété de convergence dominée des potentiels d'un noyau-fonction non-symétrique, Séminaire de Théorie du Potentiel, Paris, No. 6, Lecture Notes in Math.,906(1982), 158-202, Springer-Verlag.

[6] ——, Existence de résolvantes associées à un noyau vérifiant le principe de domination, ibid.,No. 6(1982), 203-224.

[7] ——, La balayabilité au sens fort des noyau-fonctions continues du potentiel, Nagoya Math. J.,108(1987), 145-165.

[8] ——, On the thinness of a closed set in the neighborhood of the point at infinity, 京大数理研, 講究録 1016(1997), 136-145.

[9] ——, Regularity of function-kernels and the vanishing property of potentials in the neighborhood of the point at infinity, 京大数理研, 講究録 1293(2002), 31-41.

[10] F.Hirsch, Conditions nécesaires et suffisantes d'existence de résolvantes, Z. Wahrscheinlichkeitstheorie verw. Gebiete, 29(1974), 73-85.

[11] G.A.Hunt, Markoff processes and potentials, Illinois J., Math. t.1(1957),44-94 and 316-369, t.2(1958), 151-215.

[12] M.Itô, Sur le principe relatif de domination pour les noyaux de convolution, Hiroshima Math. J., 5(1975), 293-350.

[13] ——, Positive eigen elements for an infinitesimal generator of a diffusion semi-group and their integral representations, Potential Theory Copenhagen 1979, Lecture Notes in Math., 787(1979), 164-184.

[14] ——, On weakly regular Hunt diffusion kernels, Hokkaido Math. J., 23(1981),Sp., 303-335.

[15] ——, Remarks on the regulariry for continuous function-kernels satisfying the domination principle, Nagoya Univ. preprint series No. 4(1987).

[16] M.Kishi, Inferior limit of a sequence of potentials, Proc. Japan Acad., 33(1957), Sp, 303-335.

[17] ——, Unicity principles in the potential theory, Osaka Math. J., 13(1961), 41-74.

[18] ——, Maximum principles in the potential theory, Nagoya Math. J., 23(1963), 165-187.

[19] ——, Some remarks on the existence of a resolvent, Ann. Inst. Fourier, 25, fasc, 3-4(1975), 345-352.

[20] G.Lion, Familles d'operateur et frontieres en théorie du potentiel, Ann. Inst. Fourier, 16, Fasc. 2(1966), 389-453.

[21] P.A.Meyer, Caractérisation des noyaux potentiels des semi-groupes discret, Ann. Inst. Fourier, 16, Fasc. 2(1966), 225-240.

[22] J.C.Taylor, On the existence of sub-markovian resolvent, Invent. Math. 17(1972), 85-93.

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