愛知工業大学研究報告
第３０号Ａ　平成７年

The Investigation regarding The Mean Growth Velocity Curve in Height and Weight by The Wavelet Interpolation Method

Wavelet Interpolation Method による身長・体重の平均 発育速度曲線の検討

藤　井　勝　紀
Katsunori FUJII

ABSTRACT The purpose of this study is to determine the peak height velocity age(P H V age) of the mean growth velocity curve approximated by the Wavelet Interpolation Method which author presented as an efficient procedure of analysis for this purpose. Ninety eight longitudinal data of height and weight in boys (age 6 to 17 years old) were obtained retrospectively from health examination records in 1983. When considering yearly growth distance data y { y＝F (t) } of height and weight as time series data, the growth distance curve F and growth velocity curve f (the first derivative of F) are approximated with Wavelet series by Analysing Wavelet Function of Yves. Meyer. Therefore, F (t) can be interpolated from longitudinal time series data of height and weight { (t i, y i) : i＝1, 2, 3, ……, 11 } by computer simulation (H － P UNIX Work Station). As the result, the graph y ＝ f (t) could be described clearly, and the P H V age was identified and the mean growth velocity curve of height and weight approximated was investigated. In this paper, the procedure of analysis in the study of the physical growth and development was proposed newly as the Wavelet Interpolation Method (W I M).

1. INTRODUCTION

In this study, auther consider the problem of analysing growth pattern of that as time series data for the longitudinal height and weight datas. there are considerable literatures on this problem. Tanner[1)2)] estimated the peak age(Peak Height Velocity age, P H V age)by his graphic method, and he emphasized that the P H V age could be an index of maturity speed because of the relationship between the P H V age and maturity rate. So regarding the growth pattern of height, author[3)4)] grouped the P H V age in each that age and has investigated on characteristics of the P H V age groups classified. However, because number of times measured on height were once a year, the range for a year has arose in P H V ages, and the analysis according to the determination of precise P H V age could not have done. Besides, studies that the growth pattern of height was analysed by fitting mathematical function to the growth distance curve have been done by Joossens[5)], Preece[6)], Largo[7)], Berky[8)] and logistic function of Marubini[9)] since gomperz function of Deming[10)] in 1957. In Japan, Matsuura[11)] investigate with regard to the mid-growth spurt and after-growth spurt by fitting the polynomial to the longitudinal growth distance data of individual, and Tahara and Takaishi[12)], and Ohno[13)] analyse on flattening of serial datas by fitting the spline function to the growth velocity curve. In either case, however, it is difficult to identify the peak or the peak age (especially P H V and P H V age) with accuracy. Therefore, we propose newly the Wavelet Interpolation Method(W I M) by using Analysing Wavelet of Yves. Meyer[14)15)] (Fig 1) to analyse this problem. In this method, the growth distance curve F(t) and the growth velocity curve f(t) as the first derivative of F at t were assumed to be continuos and $L^2(R)$functions. therefore, these functions can be approximated by Wavelet series. From time series datas of the mean height and weight growth, and growth velocity curves are dedecribed, and the peaks and the peak ages are identified by computer simulation. Further, the mean growth velocity curves were examined in height and weight.

2. METHOD

SAMPLE. This study is based upon a longitudinal height growth distance data of 250 japanese male subjects aged 6 to 17 years since 1972 untill 1983. These samples being derived from the Health Examination Table of public high school male junior students in Nagoya city Aichi prefecture in 1983. The Health Examination Table put down records of height, body weight, chest girth and sitting height from elementary school to high school. These records of physique items have been measured in April every year, and obligated by Ministry of Education. Samples used for this time analysis actually were extracted 98 individuals of about average height (164.0cm ～174.7cm in 17 years) which become complete without missing as the longitudinal data from subjects of 250 male students.

ANALYSIS. There is Fourier analysis in one method of analysing the time series data. Especially, " time — frequency analysis = a spectrum analysis " is used in the time series analysis etc. The characteristic of this Fourier analysis is in periodicity and similarity, and function of bounded variation F(t) is expanded to the following shape in the Fourier series for the break-uptype.

$$(1-1)\quad F(t)=\sum_{-\infty}^{\infty} c_n e^{in\pi t}$$

The discontinuity of F(t) and the discontinuity of the derivative are reflected in the attenuation order of Fourier coefficient a_n, b_n although cannot request precisely the abnormal place t as the pole value of F(t)in this method. Besides, Yves. Meyer[14)15)] is improving this and showed that arbitrary $L^2(R)$ function F made possible orthogonal expansion were done like the following shape by using Analysing Wavelet function $\psi(x)$.

$$(1-2)\quad F(t)=\sum_{j,k} a_{j,k}\psi(2^j t-k)$$

where j, k an integar.

This is called Wavelet expansion, the localization is made up by using orthogonal corollary consisted from Multiresolution Analysis.[16)] Accordingly, this Wavelet expansion is becoming extension of Fourier series which had a characteristic decided the peculiar point. Further, when the right of expansion equation (1 - 2) is possible to differentiate in each term, the first derivative f of F can also show by Wavelet expansion according to differentiating the both. The following equation consist at this

$$(1-3)\quad f(t)=\sum_{j,k} a_{j,k}2^j\psi'(2^j t-k)$$

Provided that this equation is $\psi=d\psi/dt$, $\psi(t)$ is consisted to be sufficient smooth function.

In this paper, the analysing method is to interpolated by the Wavelet analysis[17)18)]with using Analysing Wavelet function $\psi(t)$ of Yves. Meyer. The method is that when given the time series data $\{(t_i, y_i): i = 1, 2, 3, \cdots\cdots, n\}$ of n piece, we request the function $F_n(t)$, $f_n(t)$ which approximate $y=F(t)$ and the first derivative $f=dF(t)/dt$ of $F(t)$ from this data and display the graph, so this is called Wavelet interpolation.

$$(1-4)\quad F_n(t)=(\Sigma')\, a_{j,k}\,\psi(2^j t-k)$$

(1 - 5) $f_n(t) = (\Sigma')\, a_{j,k} 2^j \psi' (2^j t - k)$

Provided that (Σ') shows the total ofintegral number j , k of n piece which satisfy proper condition. When the number of the data is a few, Wavelet coefficient {a j , k ; j , k of n piece} is requested as a solution of a linear simultaneous equations of n piece which correspond to this by substituting(1-4) equation for the data of n piece given.

(1 - 6) $y_i = (\Sigma')\, a_{jk}\, \psi (2^j t_i - k)$

: i = 1, 2, ・・・・, n

y = Fn (t) means that the interpolation becomes a curve between the street and the data's point of n piece given if substitutes the Wavelet coefficient for (1-4) equation. In addition, fn (t) which approximate the growth velocity curve as well as Fn (t)is determined by substituting(1-5) equation. This curve can be displayed as the graph by computer (Fig 2).So the peak and the P H V age can be identified by this graph of fn (t) .

3.RESULTS

The interpolation problem of y = F(t)in relationship between the growth distance (y) and the age (t) was solved by using Analysing Wavelet of Yves. Meyer in each one of 98 individuals, and two examples' graph which simulated it by computer is shown in Fig 2.In this figure, the graph drawn with solid line shows growth distance curve, and which drawn with dotted line shows growth velocity curve which differentiated growth distance curve by time. As shown in Fig 2, a maximum peak(P H V) was displayed very clearly, P H V in other 96 individuals was also very clear. What the P H V age was ascertained by such as the procedure, means that it can apply the procedure to the mean growth distancies of height and weight. Then growth distancies and growth amount during a year of height and weight were calculated in 98 individuals, and mean growth distancies and velocities were done. Therefore, in this paper

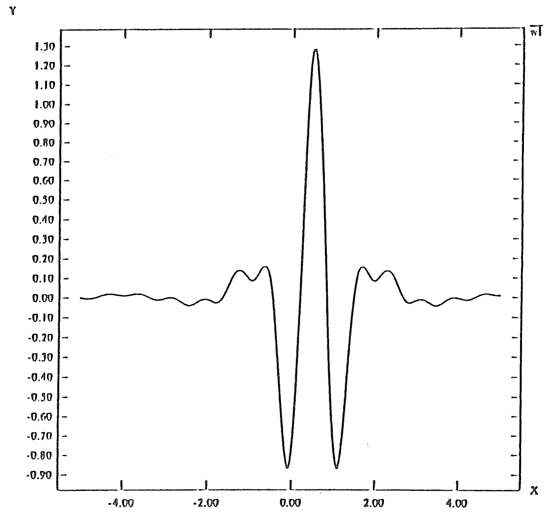


Fig 1 Yves.Meyer's Analysing Wavelet

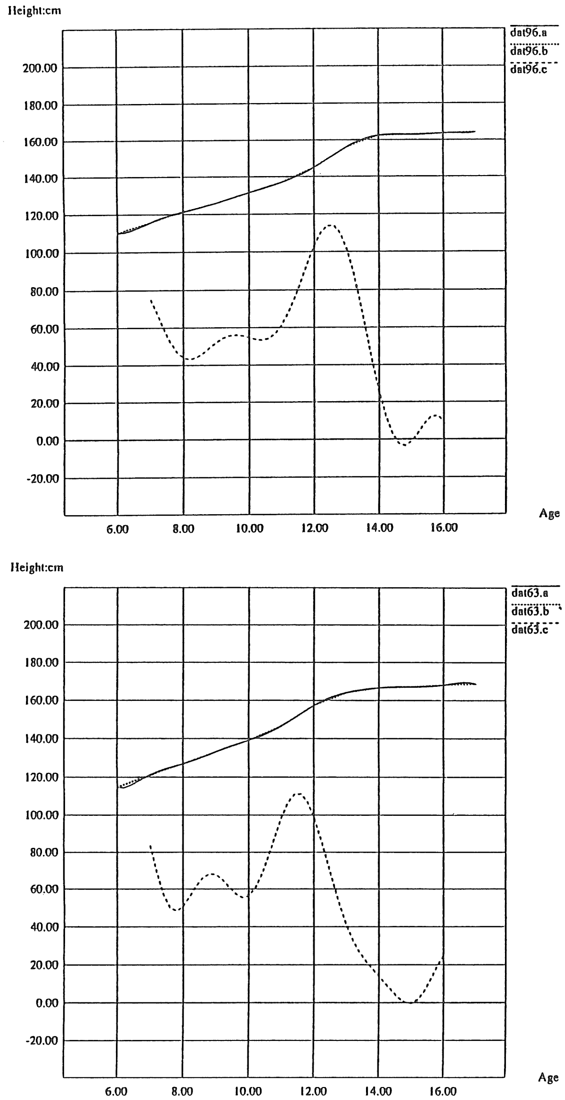


Fig 2 Height growth distance curves and velocity curves simulated by W I M

the relationship between the mean growth distance (y) and the age (t) was solved by the W I M in height and weight, and graphs of height and weight which simulated by computer is shown in Fig 3, 4. The P H V age was 12.21 years and the velocity at that time was 8.08 cm/yr in height, and in weight was 12.60 years and was 7.20 kg/yr. The peak age of mid-growth spurt was 6.68 years and the velocity at that time was 6.20 cm/yr in height, and in weight was 7.86 years and was 3.40 kg/yr.

4. DISCUSSION

Mathematical functions in the past (Logistic function, Gomperz function, Spline function and polynomial) have been investigated the precise degree and fitted the function assuming that the growth distance cueve was imagined. In these methods, however, it is difficult to explain the fitting to those curves theoretically. However, by using Meyer Wavelet with the similarity, localization and smoothness, author approach can be investigated the growth model by the simple and unified method.

By the Wavelet Interpolation Method, author examined to approximate the mean growth and velocity curve in height and weight(Fig 3,4). The attempting to approximate the mean growth curve has been done by Count[19] in 1943. Fig 5 is the graph which Count fitted three successive curves to the means for height in a cross-sectional study of Chinese children from birth to age 21. He basically fitted differential equation to the mean growth curve, and calculated as the velocity curve for which differentiated the mean growth curve approximated by it. However, he wrote that he wasted a great deal of time to calculate these procedure, and they were very complicated. Then Tanner[20] attempted to examine the growth velocity curve with simple method. The simple method is that it drew dots plotted at successive half-year age centers in accordance with raw datas of the growth distancies and slightly smoothed the drawn curve. By this graphic method, he investigated the mean growth velocity curve of height and weight based upon cross-sectional datas of 10,000 children in the 1966-67 London County Council Survey. In his paper, boys had a diminution of deceleration or relative spurt,

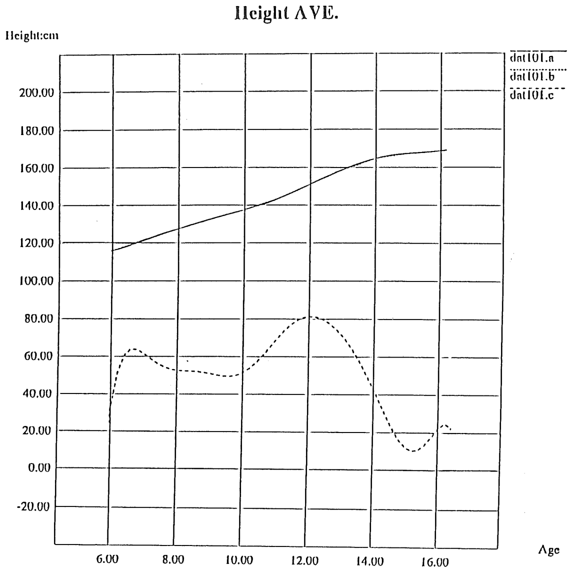


Fig 3 Mean growth distance curve and velocity curve in height approximated by W I M

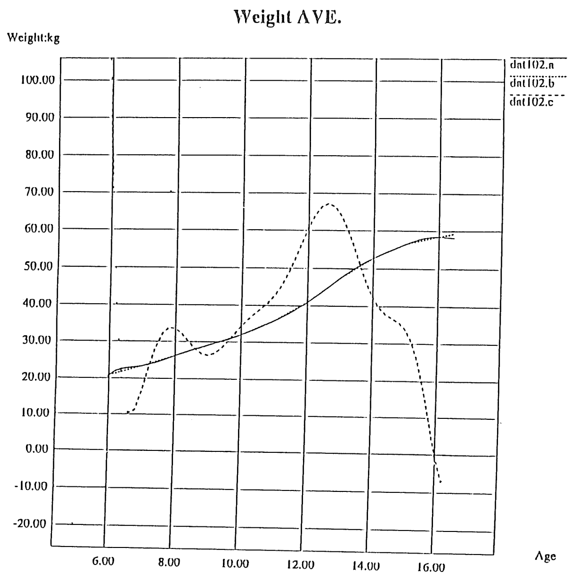
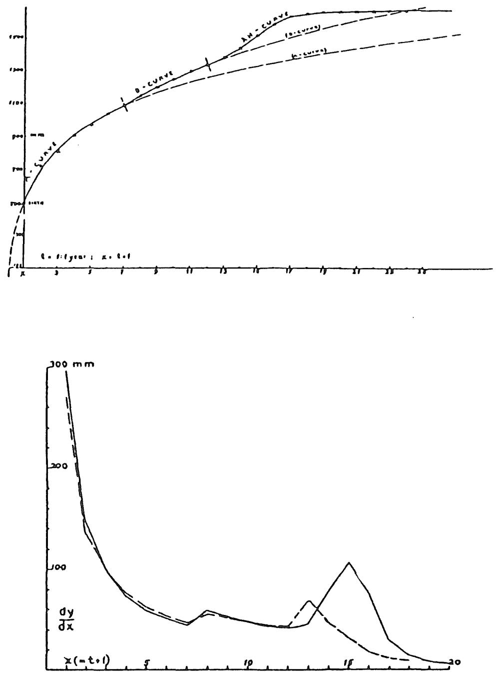


Fig 4 Mean growth distance curve and velocity curve in weight approximated by W I M

from about 6 to about 7 in height, in weight there was a clear increase of velocity occuring from 7 to 8. So he labeled Mid-Growth Spurt for these phenomena. The mid-growth spurt was ascertained in also the mean growth velocity curve of height and weight approximated by the W I M in this paper. Though Tanner and Count did not mention regarding the precise peak age of the mid-growth spurt. Author showed that the precise peak age of that was age 6.68 in height and was age 7.86 in weight, furthermore the peak velocity at the peak point of that was 6.2 cm/yr in height and was 3.4 kg/yr in weight. Of course, the precise P H V age and the velocity at that point in height and weight could be requested.

As Tanner[21] has been indicated since 1947, the existence of mid-growth spurt was ascertained in this paper. The P H V age and the peak age of mid-growth spurt, and the velocity at those point could be computed precisely by the W I M, however, the difference between the peak velocity at the P H V age in the mean growth velocity curve of height and weight approximated by the W I M, and those in the mean age of the P H V age calculated individually in each 98 occurred 2.26 cm/yr in height and 1.62 kg/yr in weight. This phenomenon is defined Phase Difference Effect, and shows the distinction of statistical procedure between cross-sectional and longitudinal datas. The statistical significance of the phenomenon is still not made clear. It should be discussed with regard to the phase difference effect in the future.

In height velocity curve approximated by the W I M, a slight onset of spurt was shown after the P H V age, but was not shown in weight velocity curve. This phenomenon of slight spurt in height velocity curve is labeled After-Growth Spurt by Matsuura[22,23]. Though this phenomenon like final spurt of last parts in height growth was shown as the result which Matsuura fitted polynomial to the height growth, was not almost shown in other literatures except Gasser's study[24,25,26]. In this paper, the after-growth spurt was displayed by the W I M also, however, the significance of the existence is not made clear and it is considered that it should be more investigated with regard to the after-growth spurt in the future.



A-curve: $y = a + bx + c \log x$,

i.e. $y = 492{\cdot}71 + 2{\cdot}26x + 682{\cdot}4 \log x$.

B-curve: $y = k(\text{A-curve}) + q$,

where $k = 1{\cdot}5, q = -547{\cdot}25$.

AH-curve: $y = v = 10^{hB}$,

$$\text{where } v = \frac{-24{\cdot}5}{1 + 10{\cdot}4365\,\text{antilog}(-6{\cdot}7529\beta)} + 33{\cdot}5$$

$$\beta = \log\left(\frac{\text{B-curve} - 1330}{10}\right)$$

$$h = -0{\cdot}003981$$

$$B = \left(\frac{\text{B-curve} - 1330}{10}\right)^{1{\cdot}66}$$

Fig 5 Count's representation of the growth distance and velocity curve fitted by mathematical function in cross-sectional data on height in Chinese children. Above, distance; below, velocity

5. CONCLUTIONS

When considering the growth process of height as the concept of time series, the relationship between growth distances（ｙ）and time (ages, ｔ), ｙ＝Ｆ（ｔ）and the ｆ（ｔ）{the first derivative of the Ｆ（ｔ）}, what is called, the problem of interpolation was solved by using Analysing Wavelet of Yves. Meyer, and the graph was displayed by the simulation. The result was concluded as the following. ＰＨＶ age was identified clearly in height and weight, the mid-growth spurt was also ascertained in both. The after-growth spurt was ascertained in height, but was not in weight. This phenomenon should be discussed in the future. Auther proposed newly as the Wavelet Interpolation Method for the procedure of analysis in this paper.

6. LITERATURE CITED

1) Tanner, J. M., Whitehouse, R. H. and Takaishi, M.:Standard from birth to maturity for height, weight, height velocity and weight velocity of British children in 1965. Arch. Dis. in Child. 41-219 :454-471, 41-220 : 613-635. 1966.
2) Tanner, J.M.:Growth at Adolescent. Blackwell Scientific Publication. 1962.
3) Fujii, K.:Growth patterns of height in boys - analysis based on velocity curve - . Bull of Aichi Institute of Technology. No 20 :39-44, 1985.
4) Fujii, K. and Matsuura, Y. :Growth patterns classified by difference of height in boys - analysis based on growth distance curve and velocity curve - . Japan J. Phys. Educ. 39 : 213-224, 1994.
5) Joossens, J. V. and Brems, Heyns. E. : High power polynomial regression for the study of distance, velocity and acceleration of growth. Growth. 39 : 535-551, 1975.
6) Preece, M. A. and Baines, M. J.: A new family of mathematical models describing the human growth curve. Annals of Human Biology. 5: 1-24. 1978.
7) Largo, R. H. et, al. : Analysis of the adolescent growth spurt using smoothing spline function. Annals of Human Biology. 5: 421-434, 1978.
8) Berkey, C. S., Reed, R. B. and Valadian, I. :Midgrowth spurt in height of Boston children. Ann. Human Biology. 10 : 25-30, 1983.
9) Marubini, E., Resele, L F., Tanner, J M. and Whitehouse, RH. : The fit of Gompertz and Logistic curves to longitudinal data during adolescence on height, sitting height and biacrominal diameter in boys and girls of the Harpenden Growth Study. Human Biology. 44 : 511-524, 1972.
10) Deming, J.:Application of the Gomperze curve to the observed pattern of growth in length of 48 individual boys and girls during the adolescent cycle of growth. Human Biology. 29 : 83-122, 1957.
11) Matsuura, Y. : A study on physical growth and development through investigating the polynomial fitted their distance curves - in the term over 6 and 18 years old. Bull of Health and Sport Sciences, Univ of Tsukuba. 14 : 211-222, 1991.
12) Tahara, K., Tatara, H., Murata, M., Takaishi, M. and Funakawa, H.:Mathematical Analysis of Adolescent Growth Acceleration Phenomenon - part 1 - Adolescentology. Vol.4:51-58, 1986.
13) Ohno, Y., Ishijima, Y. and Murata, M.:Devisal of standard physical growth curve by using spline function. Math and Sciences of statistics. 34 : 222-231. 1986.
14) Meyer, Y.:Ondelette et Operateur 1. Hermann, 1990.
15) Meyer, Y. : Ondelelettes. Hermann, 1993.
16) Mallat, S. : Multiresolution Approximations and Wavelet Ortonomal Bases of L（R）. Trans of Amer Math Soc. 315 : 69-87, 1989.
17) Frage, M. , Hunt, J. C. and Vassilicos, J. C. : Wavelets, Fractals and Fourier Transforms.

Clarendon Press, 1993.

18) Strang, G.: Wavelet Transforms versus Fourier Transforms. Bull. A. M. S. Vol. 28: 288-305, 1993.

19) Count, E. W., Growth patterns of the human physique : an approach to kinetic anthropometry. Human Biology, 15 : 1-32, 1943.

20) Tanner, J. M. and Cameron, N., Investigation of the mid-growth spurt in height, weight and limb circumferences in single-year velocity data from the London 1966-67 growth survey. Annals of Human Biology, 7 : 565-577. 1980.

21) Tanner, J. M., The morphological level of personality. Proceedings of the Royal Society of Medicine, 40 : 301-303.

22) Matsuura, Y. and Kim, M.: Analysis of physical growth by fitting the polynomial to its growth distance data - Girl's stature and body weight - . J of Korean Public Health Association. 17 : 130-148, 1991.

23) Matsuura, Y. and Kim, M. 1991. Analysis of growth by fitting the polynomial to the longitudinal growth distance data of individual - age 6 to 17 - . Research Monograph, Growth and Developmental Research, Institute of Health and Sport Sciences, Univ of Tsukuba. pp 1-153, 1991.

24) Gasser, T., Kohler, W., Muller, HG., Kneip, A., Largo, R., Molinari , L. and Prader, A. Velocity and acceleration of height growth using kernel estimation. Annals of Human Biology. 11 : 397-411. 1984.

25) Gasser, T., Muller, HG., Kohler, W., Prader, A., Largo, R. and Molinari, L. An analysis of the mid-growth and adolescent spurts of height on acceleration. Annals of Human Biology. 12 : 129-148. 1985.

26) Gasser, T., Kohler, W., Muller, HG., Largo, R., Molinari, L. and Prader, A. Human height growth : correlational and multi- variate structure of velocity and accelaration. Annals of Human Biology. 12 : 501-515. 1985.

(Received Mar. 20. 1995)